Trajectory Optimization and Control of Rigid Body Systems Through Contact

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1 Introduction

Direct methods for trajectory optimization are widely used for planning locally optimal trajectories of robotic systems. Most state-of-the-art techniques treat the discontinuous dynamics of contact as discrete modes and restrict the search for a complete path to a specified sequence through these modes. Here we outline a novel method for trajectory planning through contact that eliminates the requirement for an *a priori* mode ordering. Motivated by the formulation of multi-contact dynamics as a Linear Complementarity Problem (LCP) for forward simulation, the proposed algorithm leverages Sequential Quadratic Programming (SQP) to resolve contact constraint forces while simultaneously optimizing a trajectory and satisfying nonlinear complementarity constraints. The method scales well to high dimensional systems with large numbers of possible modes. We demonstrate the approach on two bipedal robots: planar walking with the Spring Flamingo robot, and high speed bipedal running on the FastRunner platform.

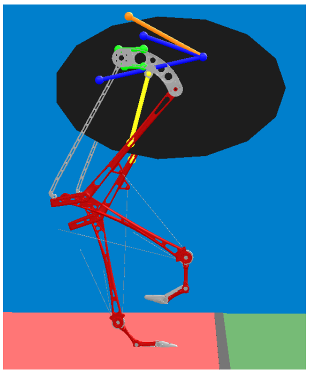

Figure 1: The bipedal FastRunner robot is designed to run at speeds of over 20 mph. Each leg has 5 degrees of freedom and multiple passive elastic elements. The legs are driven at the hip to keep the leg mass as low as possible.

2 Approach

Trajectory optimization is a powerful framework for planning locally optimal trajectories for linear or nonlinear dynamical systems. Given a control dynamical system, $\dot{x} = f(x,u)$, and an initial condition of the system $x(0)$, trajectory optimization aims to design a finite-time input trajectory, $u(t), \forall t \in [0,T]$, which minimizes some cost function over the resulting input and state trajectories. We consider the problem of designing direct trajectory optimization methods for rigid-body systems with contact. This is an essential problem for robotics which arises in any tasks involving locomotion or manipulation. The collision events that correspond with making or breaking contact, however, greatly complicate the trajectory optimization problem as they are typically modeled as discontinuous events where the system velocity changes instantaneously due to an impulsive forces.

Contact constraints formulated using the complementarity conditions, as in [3], fit naturally into the direct formulation of trajectory optimization. Rather than solving the LCP for the contact forces λ at each step, we directly optimize over the space of feasible states, control inputs, constraint forces, and trajectory durations:

$$\begin{aligned}
\underset{\{h,x_0,\dots,x_N,u_1,\dots,u_N,\lambda_1,\dots,\lambda_N\}}{\text{minimize}} \quad & g_f(x_N) + h\sum_{k=1}^{N} g(x_{k-1},u_k) && (1)\\
\text{subject to} \quad & 0 = x_k - x_{k+1} + hf(x_k,u_k) && (2)\\
& 0 \le \phi(q_k) && (3)\\
& 0 \le \lambda_{k,z} && (4)\\
& 0 \le (\mu_s \lambda_{k,z})^2 - \lambda_{k,x}^2 && (5)\\
& 0 = \phi(q_k)\lambda_{k,z} && (6)\\
& 0 = \psi(q_k,\dot{q}_k)\lambda_{k,z} && (7)
\end{aligned}$$

where $q \in \mathbb{R}^n$ is the vector of generalized coordinates, $g(\cdot,\cdot)$ is the cost function, $\phi(\cdot)$ represents the non-penetration constraint, and $\psi(\cdot,\cdot)$ is the relative velocity between two bodies in contact. This optimization program does not require specifying a mode sequence, since the contact state is embedded in the complementarity conditions of Eqns. 6 and 7. Note that the program is both nonlinear and non-convex, so we are limited to local solutions.

3 Spring Flamingo

We tested our methods on a planar simulation of the Spring Flamingo robot[2]. On Spring Flamingo, each leg has three actuated joints and two contact points on each foot. Periodic constraints were used to generate a cyclic walking gait and the trajectory was optimized for mechanical cost of transport. Where d is the total distance traveled, we write the

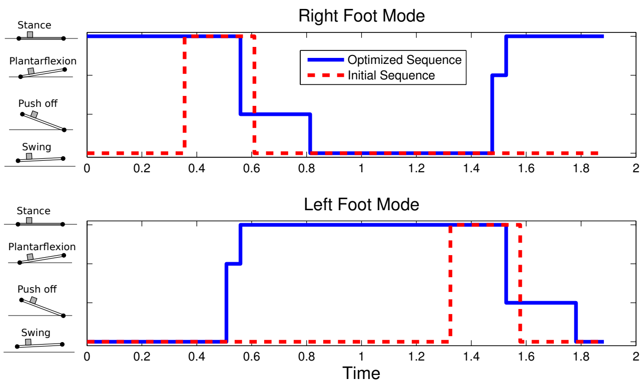

Figure 2: The optimized mode sequence of the left and right feet is plotted against time. The SQP was initialized with the significantly different sequence, demonstrating the ability of the algorithm to independently plan through contact discontinuities.

cost as:

$$g(x,u) = \frac{1}{mgd}\sum_{k=1}^{N}\sum_{i}|\dot{q}_{k,i}u_{k,i}| \tag{8}$$

Since the solutions to the NP are local, our methods discovered a wide variety of feasible gaits that satisfied the general constraints dependent on the initial condition set. For instance, given the task of finding a periodic gait that travels a specific distance, hopping motions and gaits with relatively short or long strides are possible local solutions. In particular, the input λ tape implicitly identifies the mode sequence of the initial guess. However, the solution is not restricted to the given ordering. Figure 2 shows the initial and optimized mode sequences of a particular Spring Flamingo gait. Here, the initial tape leads the trajectory to a right-left walking gait but details such as independent heel strike and heel off were identified in the optimization process.

With these linear constraints and given a nominal trajectory from Pratt's original work on the robot where the cost of transport was 0.18[1], our methods identified a periodic walking gait which reduced the cost to 0.04. This is a significant reduction in cost and is impressive in its own right, especially for a system with no passive elements to store and release energy.

4 FastRunner

This work was largely motivated by the challenges posed by the FastRunner platform shown in Figure 1. For the previous example, it is certainly possible to identify a desired mode sequence. This is a difficult task, however, for a system like FastRunner. A planar model of the robot has 13 degrees of freedom, including three articulated toe segments on each foot that can make or break contact with the ground. Additionally, There are a total of 16 unilateral joint limits, many of which are designed to be used while running at high speed. Scheduling the order of these contacts and joint limits is not practical.

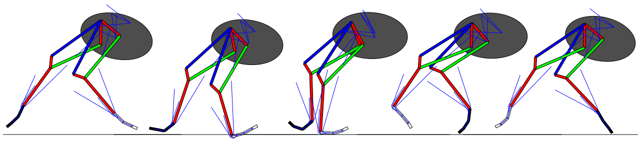

Figure 3: A generated trajectory for the FastRunner robot running at over 20 mph. The solid elements show the leg linkages and the blue lines indicate springs and tendons. Only the hip joints of the robot are actuated.

Figure 3 shows a motion sequence of an optimized periodic running gate, averaging over 20 mph. Both the leg linkages and passive elements like springs and tendons are shown in the figure. In this case, additional linear constraints were useful in guiding the NP solver away from poorly conditioned or infeasible regions. This is typical for SQP methods, where the program can be difficult to solve if the local QP is a poor estimate of the true nonlinear program.

With 22 discrete variables, there are over 4 million possible discrete modes for the FastRunner robot where each mode has a unique system of continuous dynamical equations. Despite the complexity of the system, by taking advantage of the NCP constraint formulation, our method was able to generate a locally optimal gait for FastRunner.

5 Open questions

We have also found that certain relaxations of the complementarity conditions can greatly improve the rate of convergence and reduce the likelihood of a poor, local solution. In particular, the simple relaxation $\phi(q)\lambda \leq \alpha$, where α is driven to 0 over a few passes, works well in practice. Many smoothing functions for NCPs exist and have been used to directly solve these problems and these functions may be applicable here as well.

We believe that trajectory stabilization, with a form of Model Predictive Control, is a natural extension of this work. The goal is to develop a real-time approach capable of planning a finite-horizon path with a different mode sequence than that of the nominal trajectory.

References

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