Actuated-SLIP: A Spring Mass Model with a Full Asymptotic Stability

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1 Motivation

Legged locomotion is an area of biological dynamics and bio-inspired robotics that remains to be well understood. It is still not known how animals achieve robust stability of locomotion. The canonical models and theories of the dynamics of legged locomotion differ significantly from real animals and robots is in regards to forcing (or actuation), damping, and control. Recently, there are some hip actuated leg damped spring mass models for legged locomotion have been developed. These models are more stable than the canonical energy conserving models. However, the reason behind improved stability is still not fully known. Because of the hybrid nature of legged locomotion dynamics, the role hip torque and leg damping play to stabilize center-of-mass (CoM) motion is not be as straightforward as the role active forcing and damping play to stabilize simple linear oscillators. For those hip actuated and leg damped models, their CoM linear momentum is gained and lost in directions that are orthogonal to each other during stance. However, both models are able to show an overall asymptotic stability due to momentum trading because of hybrid transitions. In addition, numerical simulation is still required for the use of these models since they are still not simple in form. Thus, how hip torque and leg damping gradually change the hybrid dynamics of legged locomotion remains to be explained and simple analytical tools of legged locomotion models with hip actuated and leg damping needs to be developed.

2 State of the Art

In the field of legged locomotion, the Spring-Loaded Inverted-Pendulum (SLIP) model was analyzed for its dynamic stability [1, 2]. It consists of a massless springy leg and a point mass. By resetting the leg landing angle to a constant value at each touchdown, it can exhibit some whole-body CoM periodic motions but are only partially stable and cannot stabilize the system energy. Then, there has been some developments of analytical solutions of SLIP [3, 4, 5] with different assumptions. One of the solutions assumed that gravity is negligible compared with the spring force and thus can be ignored during the stance phase [3]. Another solution is developed by assuming a small angle swept by the leg and a small spring compression during stance [4]. The latest analytical solution of SLIP incoporated damping during the stance and linearized the gravity around the veritical position [5]. However, these solutions are not simple to use and confirmed that SLIP is not dynamically stable.

Since then, there has been some research done on efforts to create new SLIP-based models to improve the model stability. These models have been proven to be better at resisting perturbations of the angle of velocity from the fixed point value. However they cannot stabilize the magnitude of the velocity [2, 6] because they are energy conserving.

Energy conserving models cannot describe the stability of animal or robot locomotion and suffer from one common problem: they cannot be fully asymptotically stable. Then, there have been investigations of modified spring-mass models capable of greater stability, like that of animals and robots [7, 8, 9]. Inspired by RHex [10, 11], Clock Torqued Spring-Loaded Inverted Pendulum (CT-SLIP) [7] was developed. It distinguishes itself from SLIP mainly by having a clocked governed toque at the hip and damping effects along the leg. It is fully asymptotically stable and has been successfully used to explain the robust stability of animal and robot locomotion [12]. Similar to CT-SLIP, a model called Torque-Actuated Dissipative Spring Loaded Inverted Pendulum (TD-SLIP) was presented [5]. It has a simple controlled hip torque as well as a damped leg, and is also fully asymptotically stable.

Despite the improvement of model stability, the reason is still unknown. How quantitatively hip actuation and leg damping affect the stability of CoM motion has not been explored. And simple analytical solutions of models with full asymptotically stability needs to be developed.

3 Approach

One common characteristic of energy models like CT-SLIP and TD-SLIP is that they have hip actuation and leg damping. To study the effects of hip torque and leg damping, we developed a generalized model called actuated Spring-Loaded Inverted Pendulum (actuated-SLIP) as shown in Fig. 1

Actuated-SLIP has a constant hip torque and leg damping during the stance and its flight dynamics is the same as SLIP. By explore the dynamical stability of actuated-SLIP and compared it with SLIP, we can answer understand how hip torque and leg damping affects model stability. During

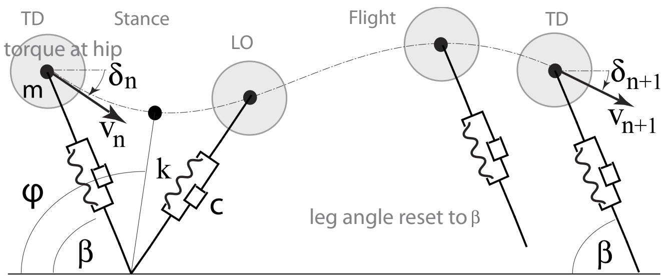

Figure 1: The general actuated-SLIP model presentation. TD stands for touchdown and LO stands for liftoff. A full stride is defined from touchdown to touchdown. The CoM velocity magnitude and direction are measured at each touchdown.

the investigation, we found hip torque and leg damping improve the model stability in a non-intuitive way. Besides, actuated-SLIP is a simple model and can be solved based on SLIP analytical approximations. And this analytical solution can be used as a simple tool for future exploration of legged locomotion dynamics and legged robot designs.

4 Discussion Outline

We intend to discuss several questions related to the effects of hip torque and leg damping as follows: 1) Does a separately stabilized stance and flight phase guarantee an overall stability of actuated-SLIP? 2) Is the stance phase of actuated-SLIP stabilized by adding hip torque and leg damping? 3) How do the hip torque and leg damping effects models stability when they are gradually increased?

Besides, we intend to discuss how can we develop the analytical solution of actuated-SLIP with a form not complicated but able to have a qualitatively similar prediction about model stability compared with numerical simulation. And what is potential usage of this analytical solution.

5 Preferred Format

15 minutes talk, 5 minutes talk or poster.

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