Design of stable walking gaits for biped robots with several underactuated degrees of freedom

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1. Introduction

- Motivation: Theoretically, dynamic walking gaits are periodic solutions that can exist either as a natural response of the system or by the use of feedback control. Analytically, finding these solutions is certainly challenging, and the complexity of the problem increases considerably when a mixture in between passive and actuated joints is considered, e.g. underactuated robots.

- Solution: Virtual holonomic constraints (VHC) allows a tractable analytical and practical approach to achieve stable limit cycle motions.

- Difficulties: There is a lack of a systematic procedure to find these VHC for systems with underactuation higher than one. This can be ease as shown in the example below.

- Model: The model represents a planar biped walker with two symmetric legs and upper torso.

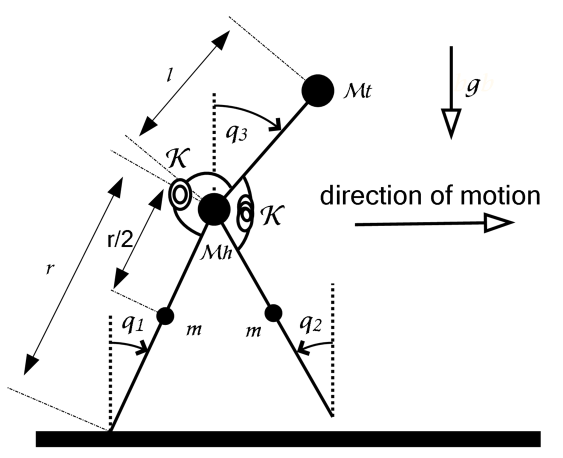

Figure: **Biped.** Schematics of the biped in the sagittal plane and level ground.

- The dynamics during the stance phase is given by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \begin{bmatrix} u \\ -u \\ 0 \end{bmatrix}.$$

- The instantaneous jump in the values of the states (post-impact) can be computed as follows

$$\begin{bmatrix} q^+ \\ \dot{q}^+ \end{bmatrix} = \Delta \begin{bmatrix} q^- \\ \dot{q}^- \end{bmatrix} \Rightarrow q^- \in \Gamma^-,\ q^+ \in \Gamma^+.$$

- The impact condition is

$$\Gamma^+ = \Gamma^- = cos(q_1) - cos(q_2) = 0.$$

- Objective: The main technical objective is to show a procedure for gait synthesis based on VHC for systems with underactuation degree higher than one.

2. Gait Synthesis

The gait synthesis approach consists of finding a set of time independent geometric functions, which uniquely describe the instantaneous postures of the robot along the gait:

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \begin{bmatrix} \theta(t) \\ \phi_2(\theta(t)) \\ \phi_3(\theta(t)) \end{bmatrix}, \quad \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \phi_2'(\theta)\dot{\theta} \\ \phi_3'(\theta)\dot{\theta} \end{bmatrix}, \quad \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} = \begin{bmatrix} \ddot{\theta} \\ \phi_2''(\theta)\dot{\theta}^2 + \phi_2'(\theta)\ddot{\theta} \\ \phi_3''(\theta)\dot{\theta}^2 + \phi_3'(\theta)\ddot{\theta} \end{bmatrix},$$

where $\theta \in [\theta^+, \theta^-]$, representing the initial and final stance-leg angles. With this scheme the stance leg becomes a form of trajectory generator, which dynamics is defined as

$$\begin{aligned} \alpha_1(\theta)\ddot{\theta} + \beta_1(\theta)\dot{\theta}^2 + \gamma_1(\theta) &= u, \\ \alpha_2(\theta)\ddot{\theta} + \beta_2(\theta)\dot{\theta}^2 + \gamma_2(\theta) &= -u. \\ \alpha_3(\theta)\ddot{\theta} + \beta_3(\theta)\dot{\theta}^2 + \gamma_3(\theta) &= 0, \end{aligned}$$

Notice that the previous set of differential equations are linear in $\ddot{\theta}$, $\dot{\theta}^2$, and u, giving the solution

$$\begin{bmatrix} \ddot{\theta} \\ \dot{\theta}^2 \\ u \end{bmatrix} = \begin{bmatrix} \alpha_1(\theta) & \beta_1(\theta) & -1 \\ \alpha_2(\theta) & \beta_2(\theta) & 1 \\ \alpha_3(\theta) & \beta_3(\theta) & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\gamma_1(\theta) \\ -\gamma_2(\theta) \\ -\gamma_3(\theta) \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}.$$

MAIN CONTRIBUTION

Given the relation between the derivatives

$$\ddot{\theta}(t) = \frac{1}{2}\frac{d}{d\theta}\left(\dot{\theta}^2(t)\right) \Rightarrow D_1 = \frac{1}{2}\frac{d}{d\theta}(D_2),$$

we find that

$$2D_1 = \frac{\partial D_2}{\partial \theta} + \frac{\partial D_2}{\partial \phi_2}\phi_2' + \frac{\partial D_2}{\partial \phi_3}\phi_3' + \frac{\partial D_2}{\partial \phi_2'}\phi_2'' + \frac{\partial D_2}{\partial \phi_3'}\phi_3'' + \frac{\partial D_2}{\partial \phi_2''}\phi_2''' + \frac{\partial D_2}{\partial \phi_3''}\phi_3''',$$

yielding the differential equation

$$\phi_3''' = f(\theta, \phi_2, \phi_3, \phi_2', \phi_3', \phi_2'', \phi_3'', \phi_2'''),$$

for the torso's trajectory ϕ_3 as a function of the legs i.e., θ and ϕ_2. It can be solved given the function ϕ_2, and the initial condition vector,

$$\zeta = [\theta^+, \phi_3^+, \phi_3'^+, \phi_3''^+] \in \mathbb{R}^{4\times 1},$$

In this form, the problem of finding the two functions ϕ_i, gets reduced to the search for ϕ_2 to solve ϕ_3'''.

3. Optimization procedure to find periodic gaits

1. The motion of the swing leg ϕ_2 can be defined by an arbitrary function C^3 smooth, e.g. Bézier polynomial.
2. Choose a vector of initial conditions and parameters such that
$$\chi_0 = \left[\theta^+, \phi_2^+, \phi_3^+, \dot{q}_1^+, \dot{q}_2^+, \dot{q}_3^+\right] \in \Gamma^+.$$
3. Calculate $\phi_3''^+$ from dynamics of the robot.
4. Define $\theta^- = -\theta^+$, to solve the differential equation ϕ_3''', if the solution is not well-defined for the whole interval go back to the first step.
5. Once the solution $\phi_3(\theta)$ has been found, compute $\dot{\theta}$, i.e.,
$$\dot{\theta}^- = \sqrt{D_2(\theta^-)} = \dot{q}_1^-.$$
6. Use the end-values of the solution from the previous steps to define the vector
$$\chi_\star^- = \left[\theta^-, \phi_2^-, \phi_3^-, \dot{q}_1^-, \dot{q}_2^-, \dot{q}_3^-\right] \in \Gamma^-.$$
7. Apply the impact equation, i.e.
$$\chi_\star^+ = \Delta \cdot \chi_\star^-.$$
8. Verify that
$$\|\chi_0 - \chi_\star^+\| \approx 0.$$
If such an equality does not comply, adjust the vector χ_0 and repeat all the process.
9. This procedure can be formulated to additionally attain to minimize a cost function
$$J = \frac{1}{L_{step}} J_0(u),$$
where $L_{step} = 2r\sin(\theta^+)$ is the step length, and $J_0(u)$ is an integral of the absolute power needed to generate this trajectory, i.e.,
$$J_0(u) = \int_0^{T_e} \left|\left(\dot{q}_{1\star}(t) - \dot{q}_{2\star}(t)\right) u\right| dt,$$

4. Simulations

- Gait: One gait found by this procedure has the solution's vector given by
$$\chi_\star \approx [-0.2503, 0.2503, 0.3131, 1.2512, 0.4956, -0.2695],$$
$q_1 \in [-0.2503, 0.2503]$, $T_e = 0.4878$ sec, $K = 23.9770$ N/m. The numerical value of energy calculated for this gait is 21.16 W/m. The constraint functions $\phi_2(\theta)$, $\phi_3(\theta)$, and the control signal $u(\theta)$ are shown below.

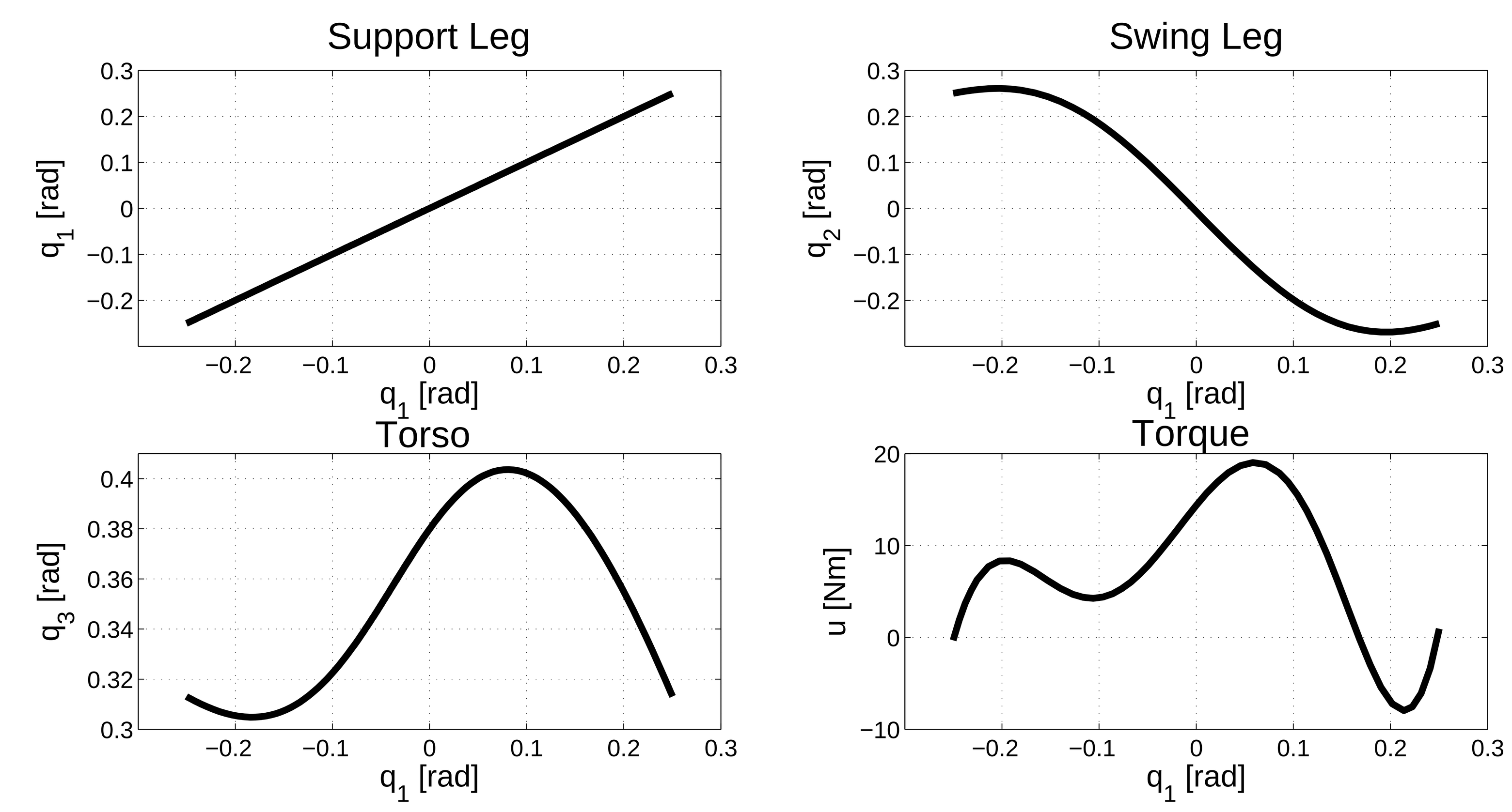

Figure: Trajectories as functions of q_1 with initial conditions $\chi_\star$. The motion goes from left to right.

- Control: The gait is stabilized by feedback control, and applying the technique know as hybrid transverse linearization proposed by Prof. A. Shiriaev to the case of systems with several passive links.

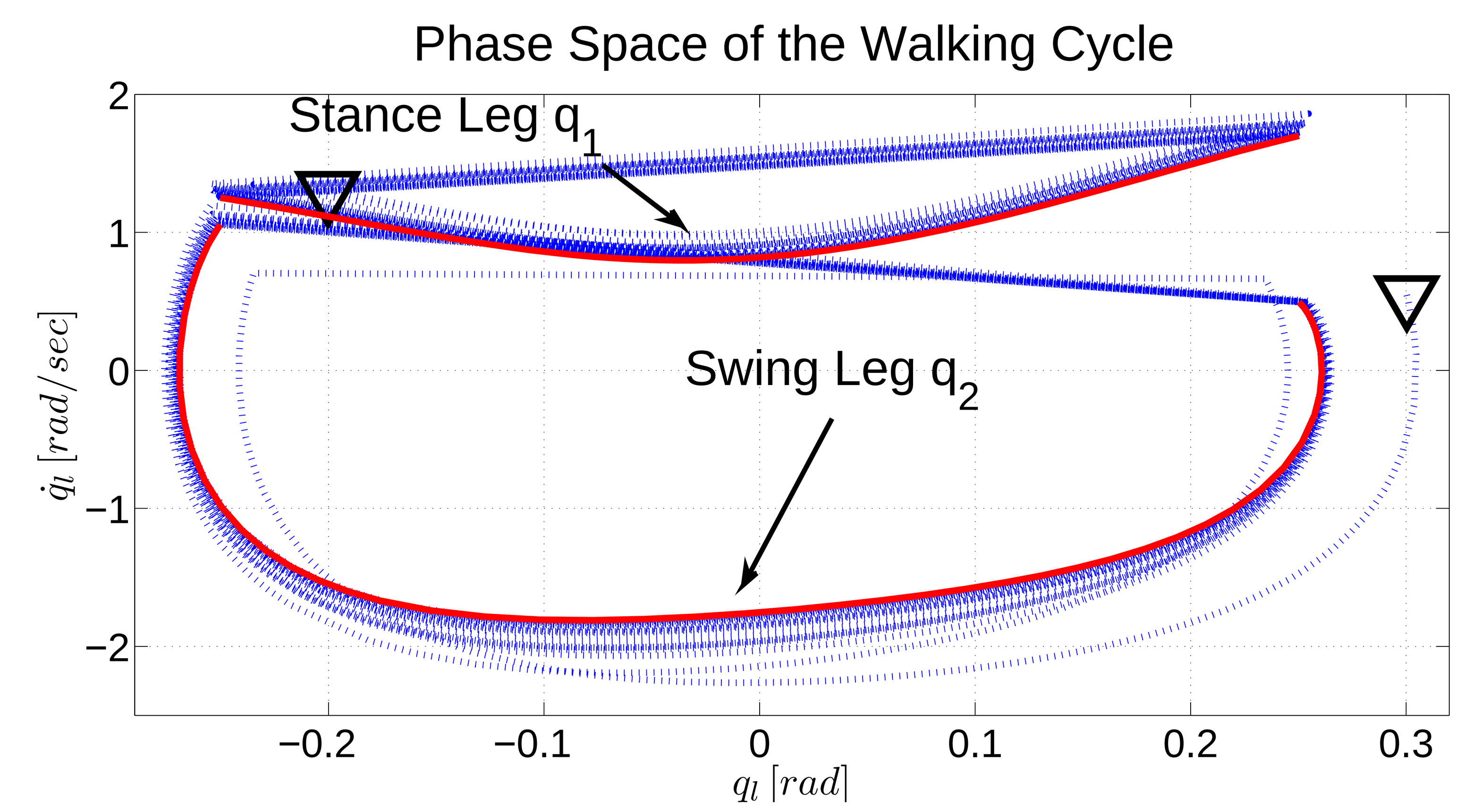

Figure: Walking gait in the phase space q_l vs $\dot{q}_l$, i.e. stance and swing legs positions and velocities. The red line represents the nominal gait, and $\triangledown$ points out the initialization of the simulation.