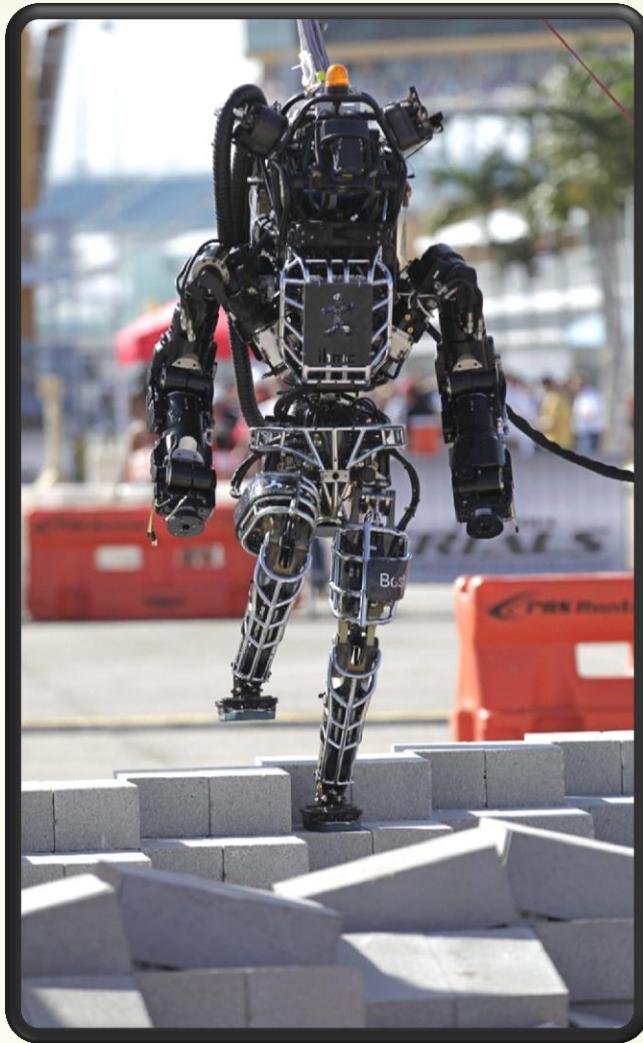


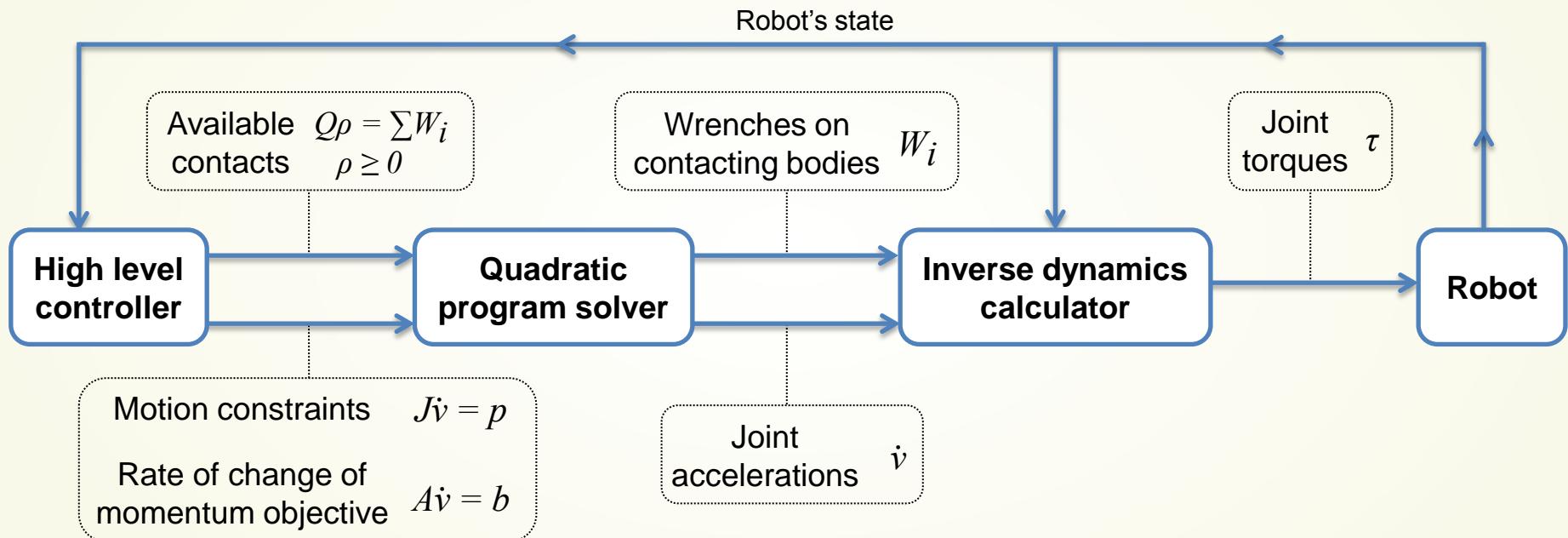


Momentum-based control framework, capturability-based walking control: application to the humanoid robot Atlas

Presented by Tingfan Wu and Sylvain Bertrand

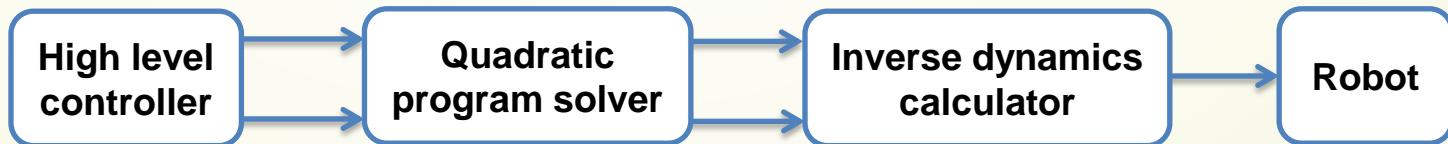


Whole-body motion control framework



Motion Constraints – Instantaneous Goals

- Examples
 - ☺ Maintain balance – Keep CoM between feet
 - ☺ Move my hand closer to the target
 - ☹ Step on rock – instantaneous move!
- Divide long term goal to short ones
 - Trajectory planner (eg. PID controller, swing planner)



Temporal layers	seconds	166Hz	166Hz	1kHz
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Linear Constraints on Joint Acceleration

$$\min_{\dot{\nu}} \sum w_i \|M_i \dot{\nu} - p_i\|^2$$

$$s.t \quad M_j \dot{\nu} = p_j$$

- Connect with desires linearly, e.g.

$$\dot{\nu}_k = \nu_{d,k}^{\cdot}$$

- Change of reference frame via Jacobian

$$J\nu = \dot{x}_d$$

$$J\nu + J\dot{\nu} = \ddot{x}_d$$

$$J\dot{\nu} = \ddot{x}_d - J\nu$$

Linear Constraints on Joint Acceleration

$$\min_{\dot{v}} \sum w_i \|M_i \dot{v} - p_i\|^2$$

$$s.t \quad M_j \dot{v} = p_j$$

- Connect with desires linearly, e.g.

$$\dot{v}_k = v_{d,k}$$

- Change of reference frame via Jacobian

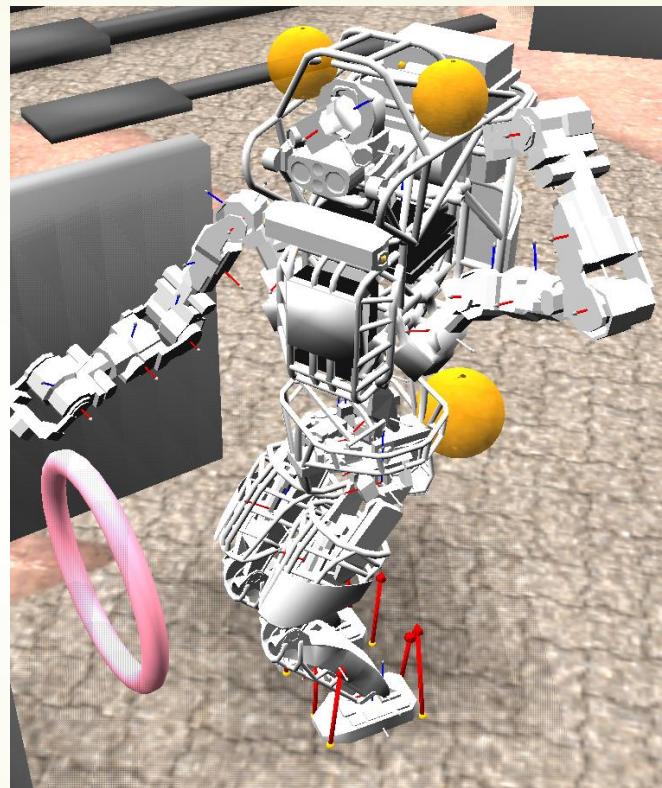
$$J\dot{v} = \ddot{x}_d - Jv$$

- Link to velocity / position goals with PD controller

$$\dot{v}_k = k_p(x_d - x) + k_d(\dot{x}_d - \dot{x})$$

Constraints / Desires of Atlas

- Chest rotation
 - Joint space goal
- Scratch head
 - Spatial goal in body
- Reach the torus
 - spatial goal in world



Centroidal Momentum

- Single Link- Angular and Linear Momentum

$$h = \begin{bmatrix} k \\ l \end{bmatrix} = \begin{bmatrix} I\omega \\ m\dot{x} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & m\mathbf{1} \end{bmatrix} \begin{bmatrix} \omega \\ \dot{x} \end{bmatrix}$$

- Multiple Links - Centroidal Momentum

- sum of link momenta expressed in centroidal frame
 - A **linear** function of joint velocities

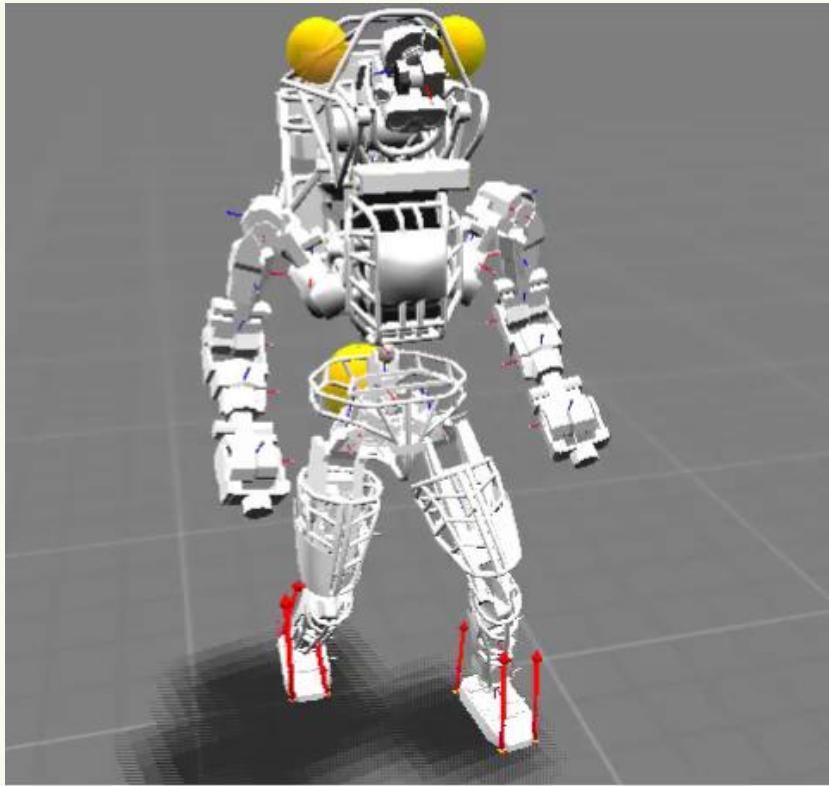
$$h = A(q)\nu$$

- Enforce $\dot{h} = \dot{h}_d$ with constraint

$$\dot{A}\nu + A\dot{\nu} = \dot{h}_d$$

$$A\dot{\nu} = \dot{h}_d - \dot{A}\nu$$

Example of Controlling Centroidal Momentum



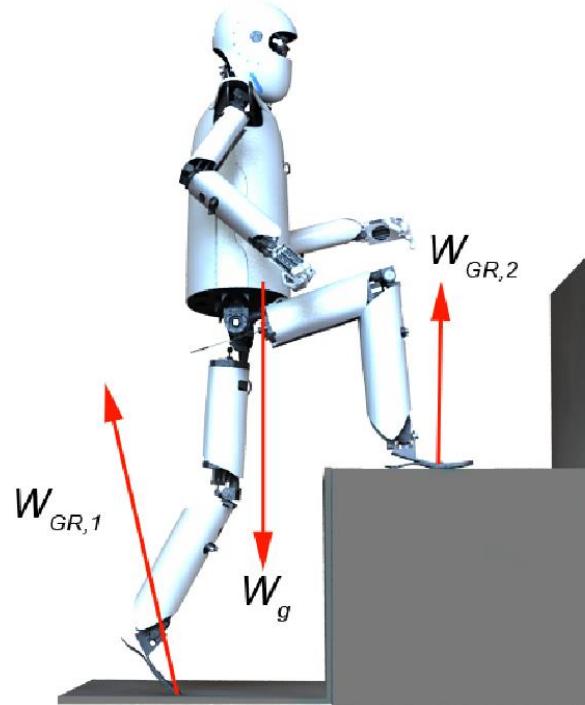
Constraint on External Wrenches

- Newtown/Euler law

$$\dot{h} = \sum \text{external wrench}$$

$$\dot{h} = \dot{A}\nu + A\dot{\nu} = \sum W_{ext,i}$$

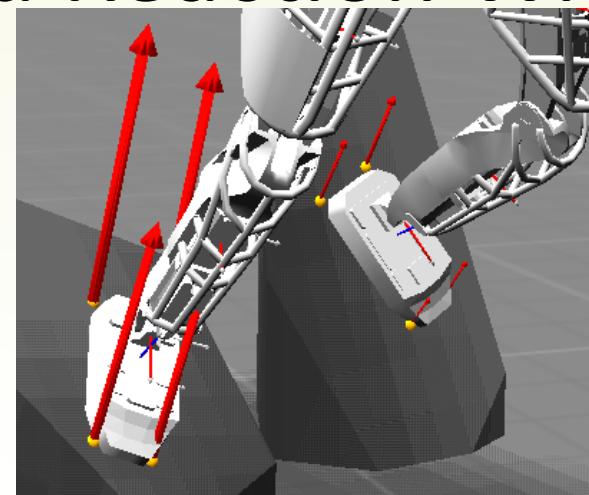
$$A\dot{\nu} = -\dot{A}\nu + \sum W_{GR,i} + W_g$$



$$\sum_i W_{ext,i} = \sum_i W_{GR,i} + W_g$$

Constraint on Ground Reaction Wrench

Construct force in pyramid approximation
of friction cone [Pollard and Reitsma, 2001]

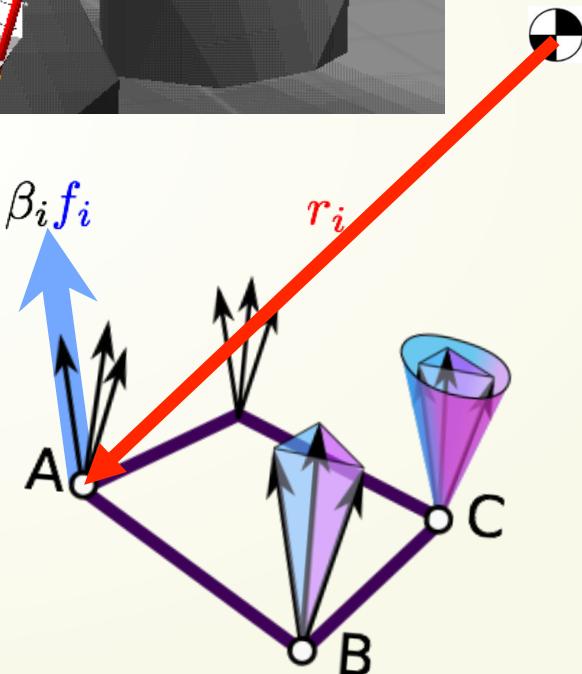


$$\text{force } f_i = \beta_i \rho_i, \rho_i \geq 0$$

$$\text{torque } \tau_i = r \times f_i$$

$$\text{wrench } w_i = \begin{bmatrix} \beta_i \\ r_i \times \beta_i \end{bmatrix} \rho_i = q_i \rho_i$$

$$w_{GR} = \sum w_i = \sum q_i \rho_i = Q \rho$$



Put It All Together

minimize

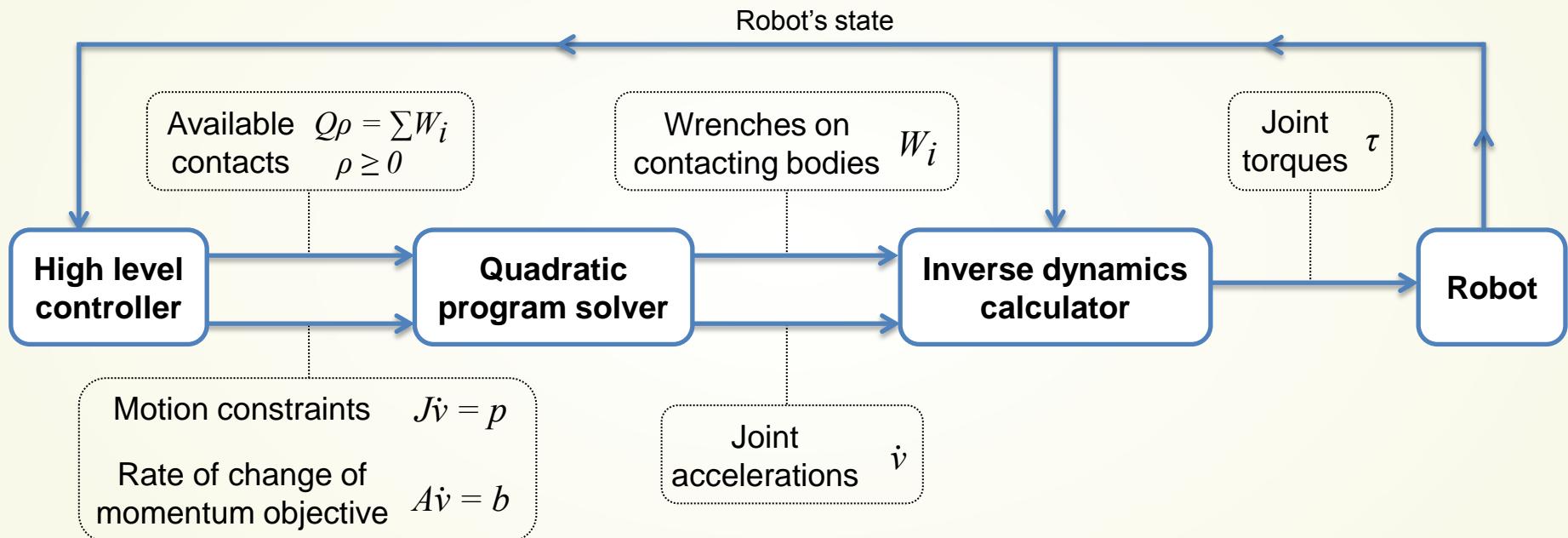
$$\min_{\dot{v}, \rho} \sum w_i \|M_i \dot{v} - p_i\|^2 + w_{\dot{v}} \|\dot{v}\|^2 + w_{\rho} \|\rho\|$$

s.t

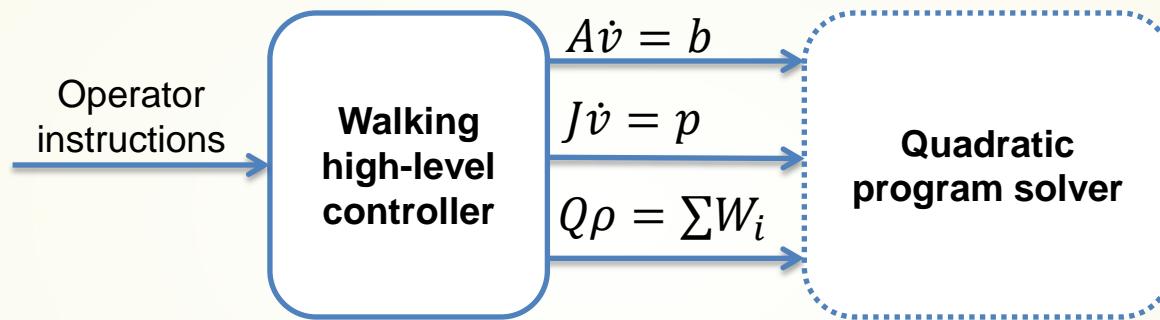
$$\rho \geq 0$$

$$A\dot{v} = -\dot{A}v + W_g + Q\rho$$

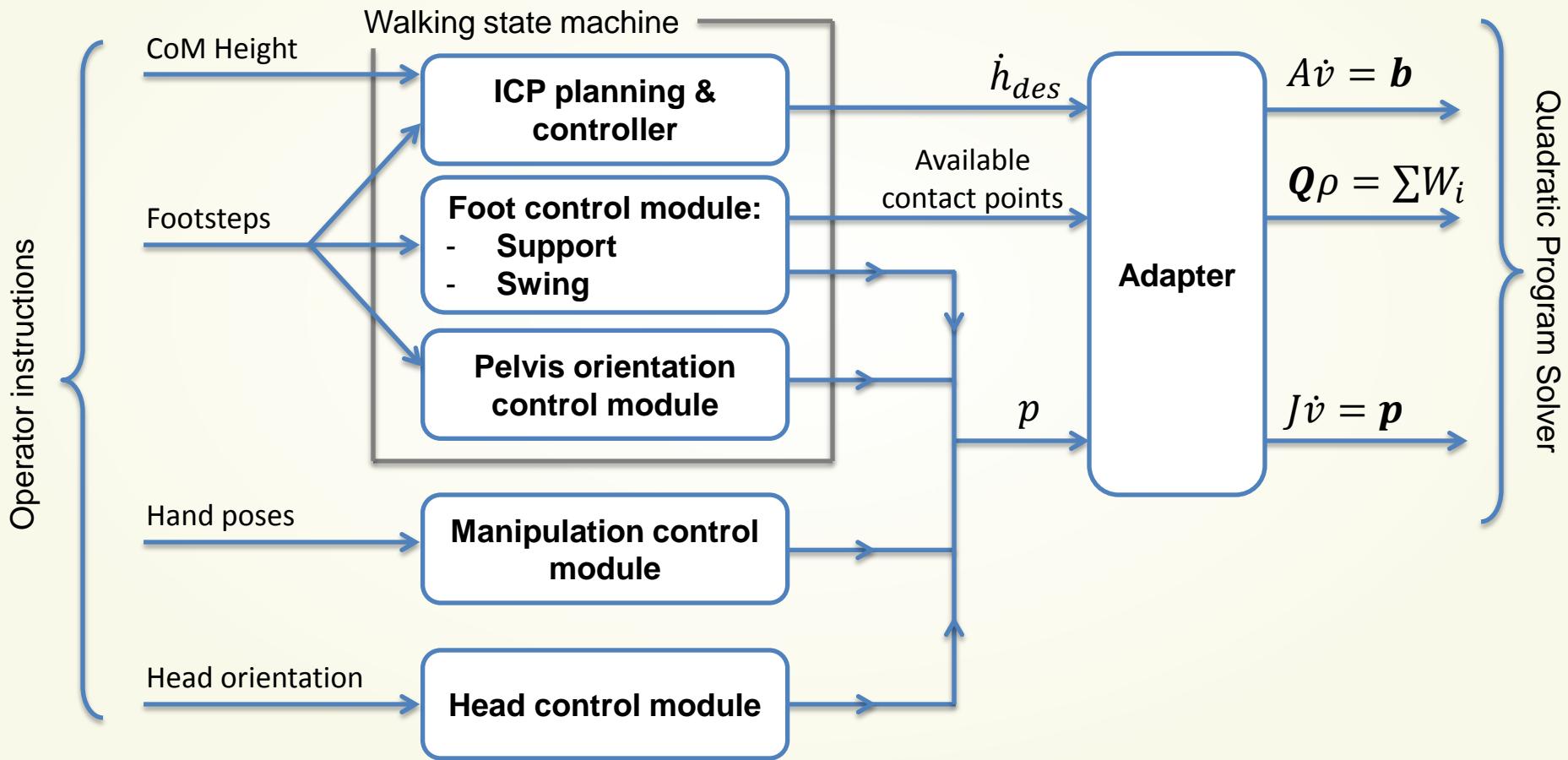
Whole-body motion control framework



Walking high-level controller



Walking high-level controller



Pelvis Orientation Control Module

- Minimum jerk trajectory generator:

$$\theta_{\text{pelvis}}^{\text{ref}}, \omega_{\text{pelvis}}^{\text{ref}}, \dot{\omega}_{\text{pelvis}}^{\text{ref}}$$

- PD controller:

$$\dot{\omega}_{\text{pelvis}}^{\text{des}} = \dot{\omega}_{\text{pelvis}}^{\text{ref}} + K_p(\theta_{\text{pelvis}}^{\text{ref}} - \theta_{\text{pelvis}}) + K_v(\omega_{\text{pelvis}}^{\text{ref}} - \omega_{\text{pelvis}})$$

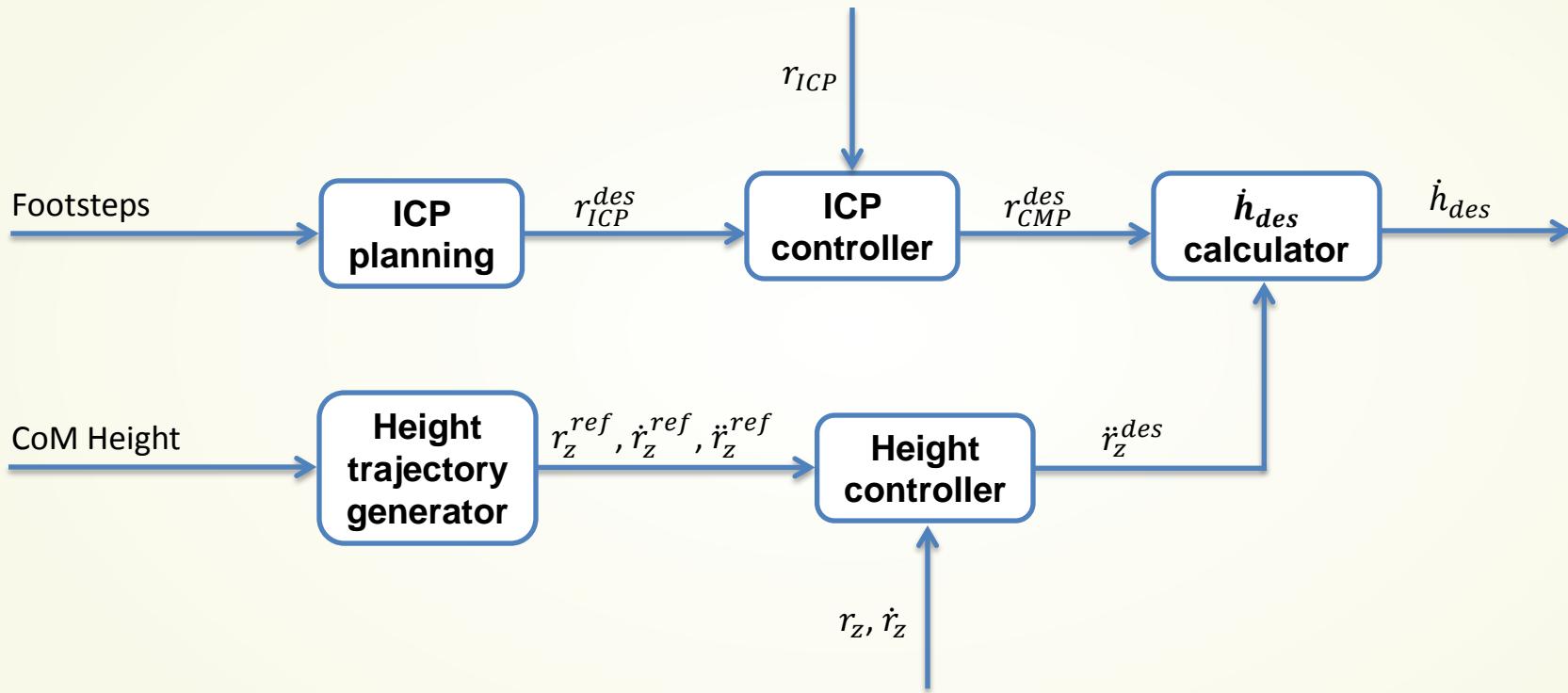
- Motion constraint:

$$SJ\dot{v} = Sp$$

With:

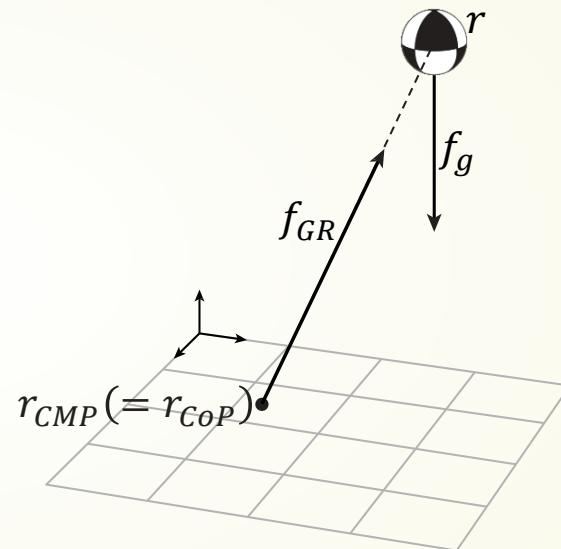
$$p = \begin{bmatrix} \dot{\omega}_{\text{pelvis}}^{\text{des}} \\ 0_{3 \times 1} \end{bmatrix} - jv$$
$$S = (I_{3 \times 3} \quad 0_{3 \times 3})$$

ICP Planning & Controller



ICP Overview

- Point mass:
 - $r_{CMP} = r_{CoP}$
 - CoP must be inside support polygon
 - CMP inside the support polygon too



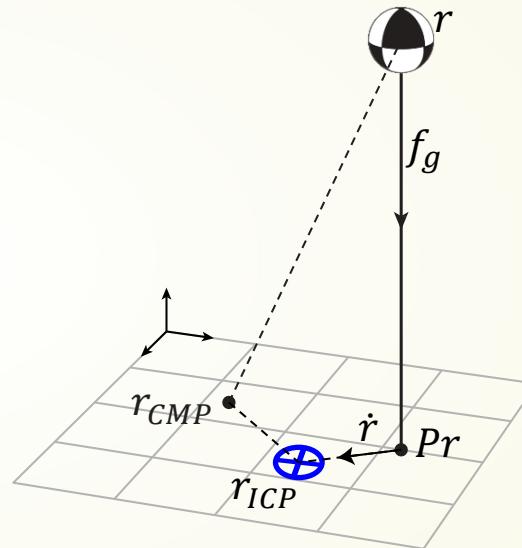
ICP Overview

- Instantaneous Capture Point:

$$r_{ICP} = Pr + \frac{1}{\omega_0} \dot{r}$$

with:

$$\omega_0 = \sqrt{\frac{g}{r_z}}$$



ICP Overview

- Instantaneous Capture Point:

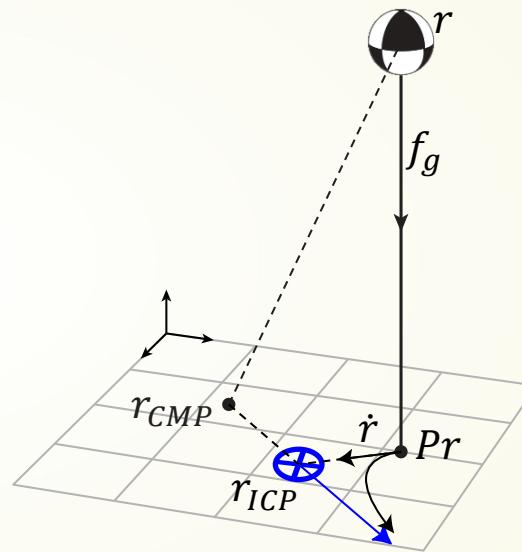
$$r_{ICP} = Pr + \frac{1}{\omega_0} \dot{r}$$

with:

$$\omega_0 = \sqrt{\frac{g}{r_z}}$$

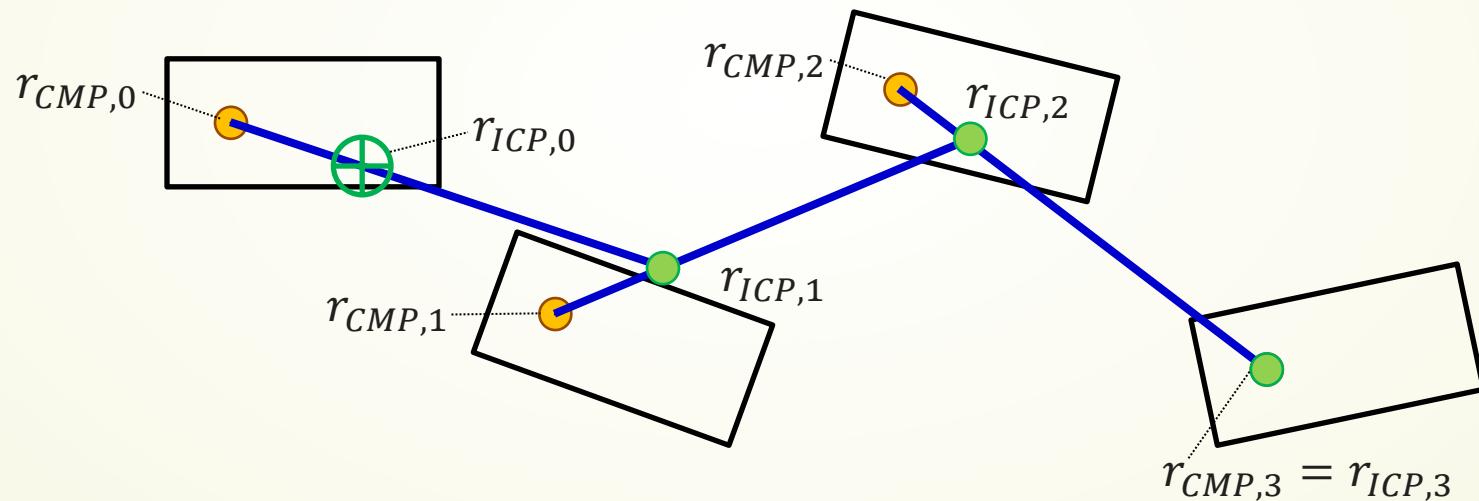
- Dynamics for constant r_z :

$$\dot{r}_{ICP} = \omega_0(r_{ICP} - r_{CMP})$$



ICP Planning (Simpler case)

- Plan r_{CMP} for single supports (shift instant. In double support)
- Find with a recursion r_{ICP} at the beginning of single support
- Generate desired trajectory for the ICP



Englsberger, J. et al. "Three-dimensional bipedal walking control using Divergent Component of Motion", IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), 2013, 2600-2607

ICP Controller & \dot{h}_{des} Calculation

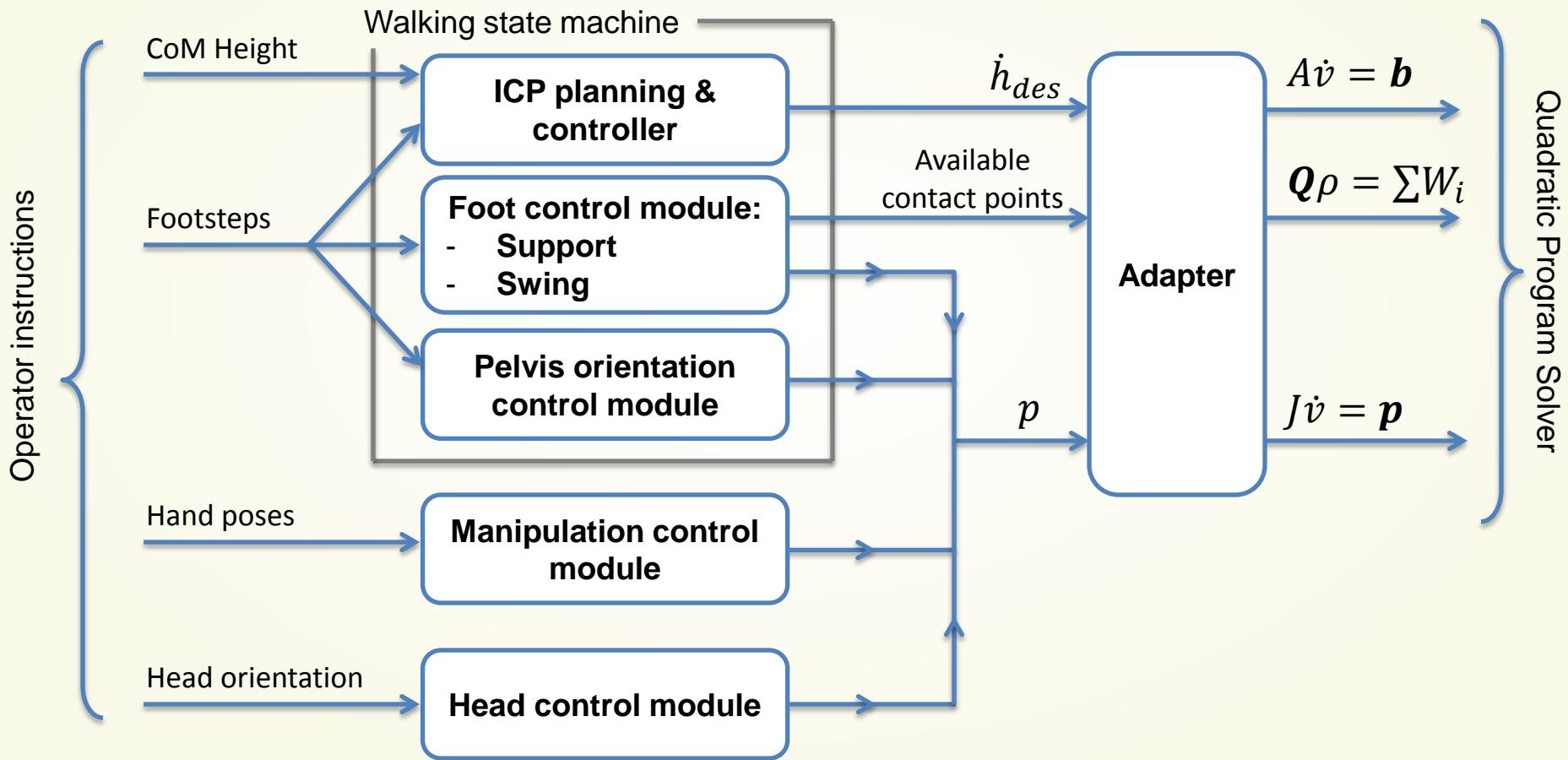
- ICP controller:

$$r_{CMP}^{des} = r_{ICP} + \frac{K_p}{\omega_0} (r_{ICP}^{des} - r_{ICP}) - \frac{1}{\omega_0} \dot{r}_{ICP}^{des}$$

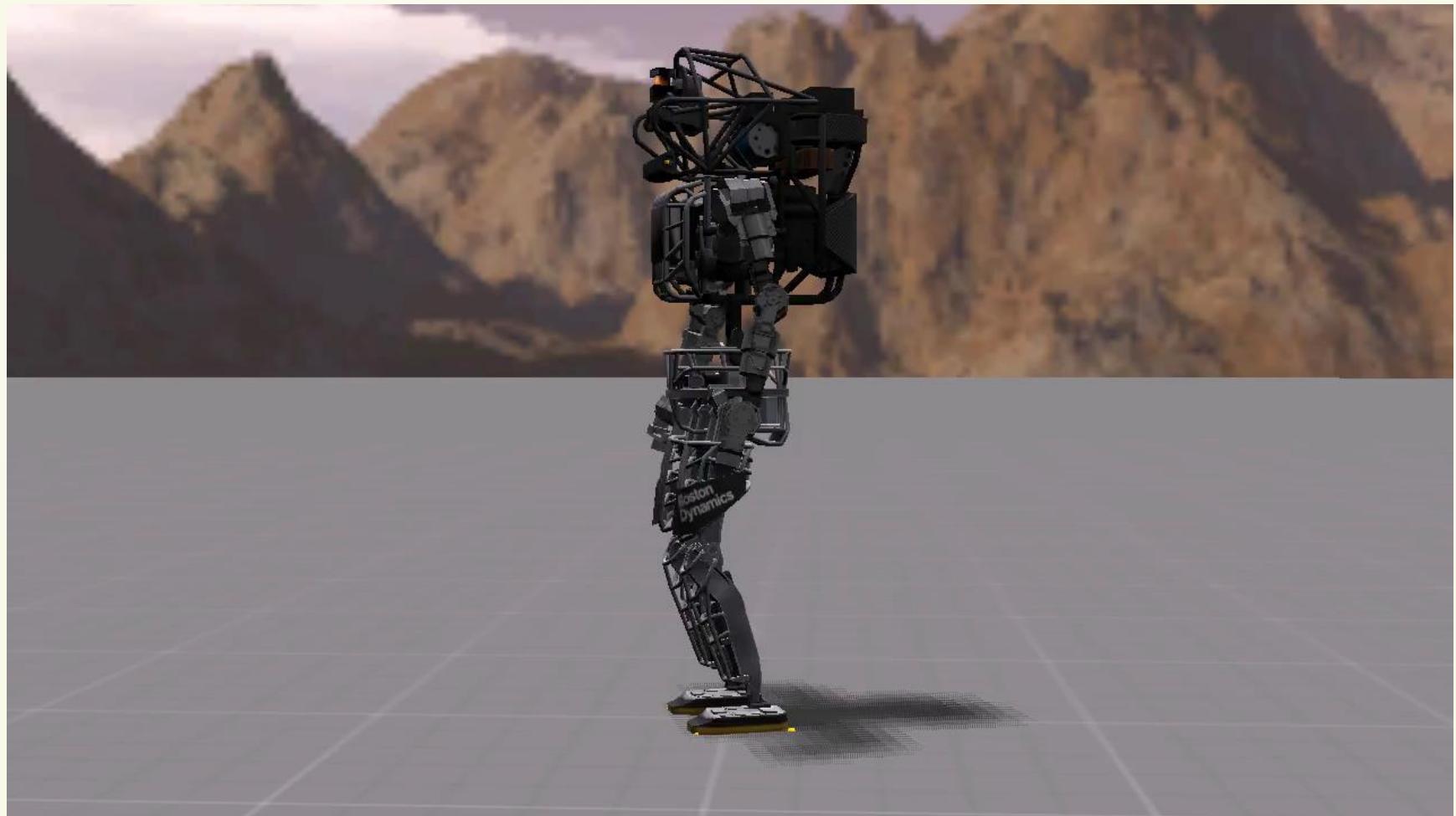
- Compute $\dot{h}_{des}()$:

$$\dot{h}_{des} = m \begin{bmatrix} \omega_0^2 (r_x - r_{CMP,x}) \\ \omega_0^2 (r_y - r_{CMP,y}) \\ \ddot{r}_z^{des} \end{bmatrix}$$

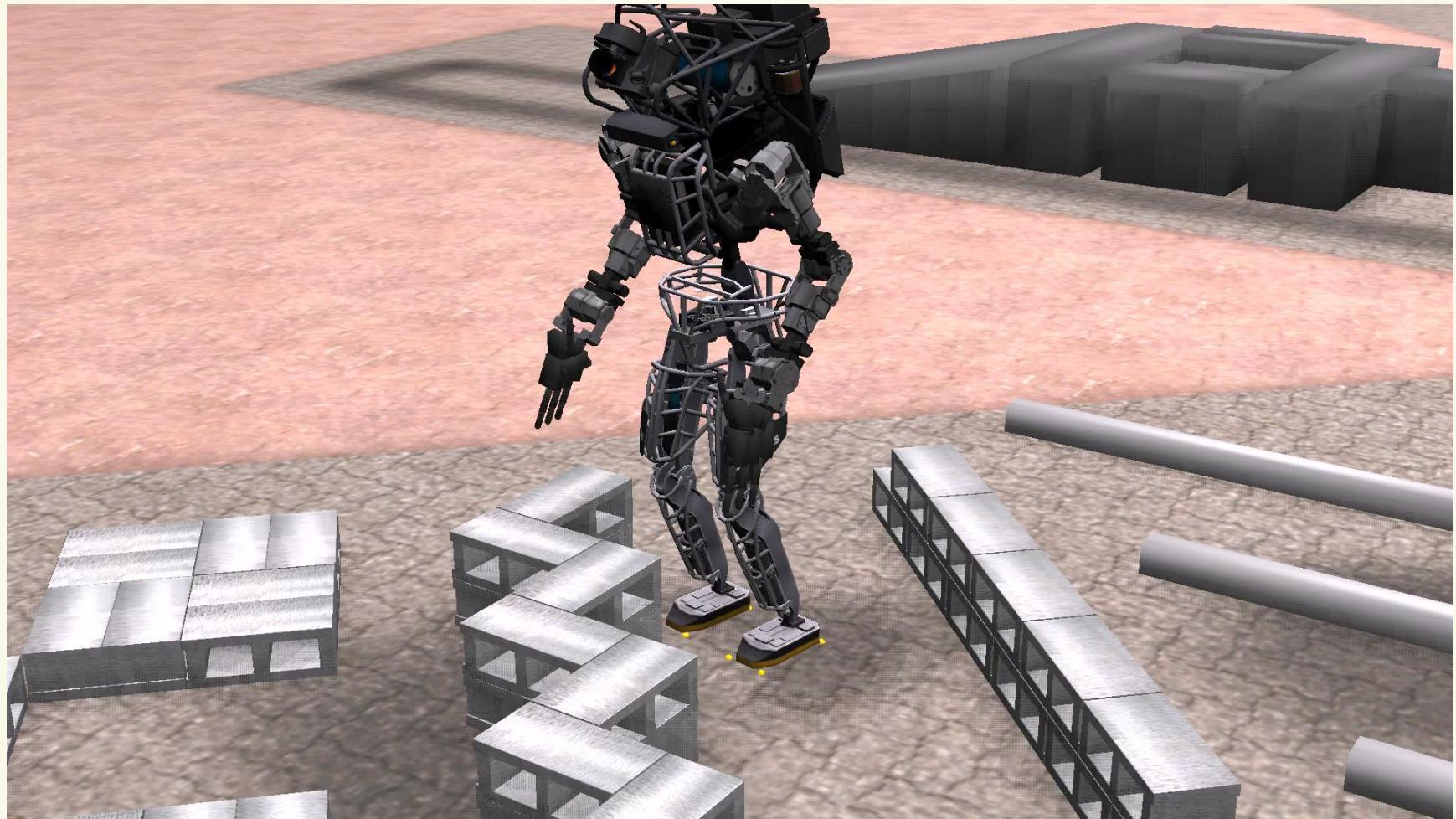
Walking high-level controller

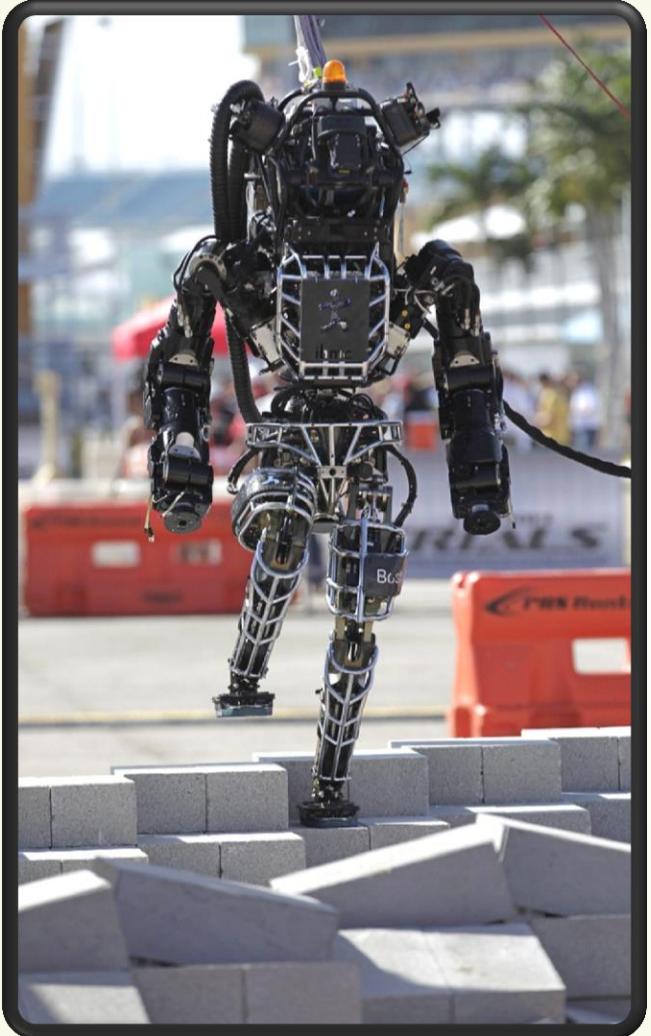


Virtual Atlas Walking



Virtual Atlas Walking





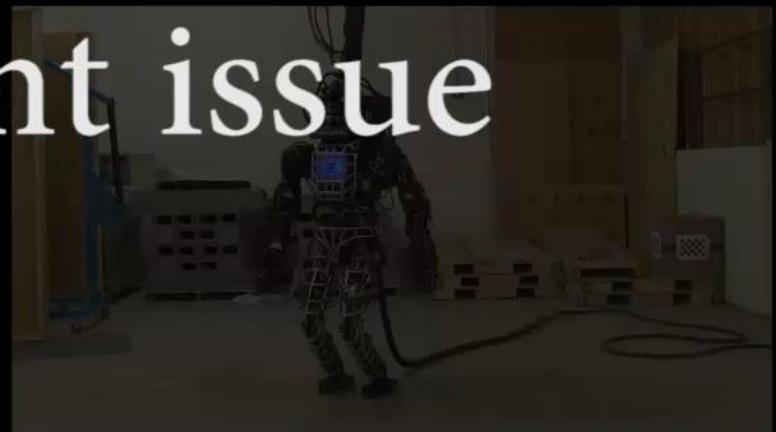
IMPLEMENTATION ON THE REAL ROBOT

Issues: Shakies

Robot shaking



Persistent issue



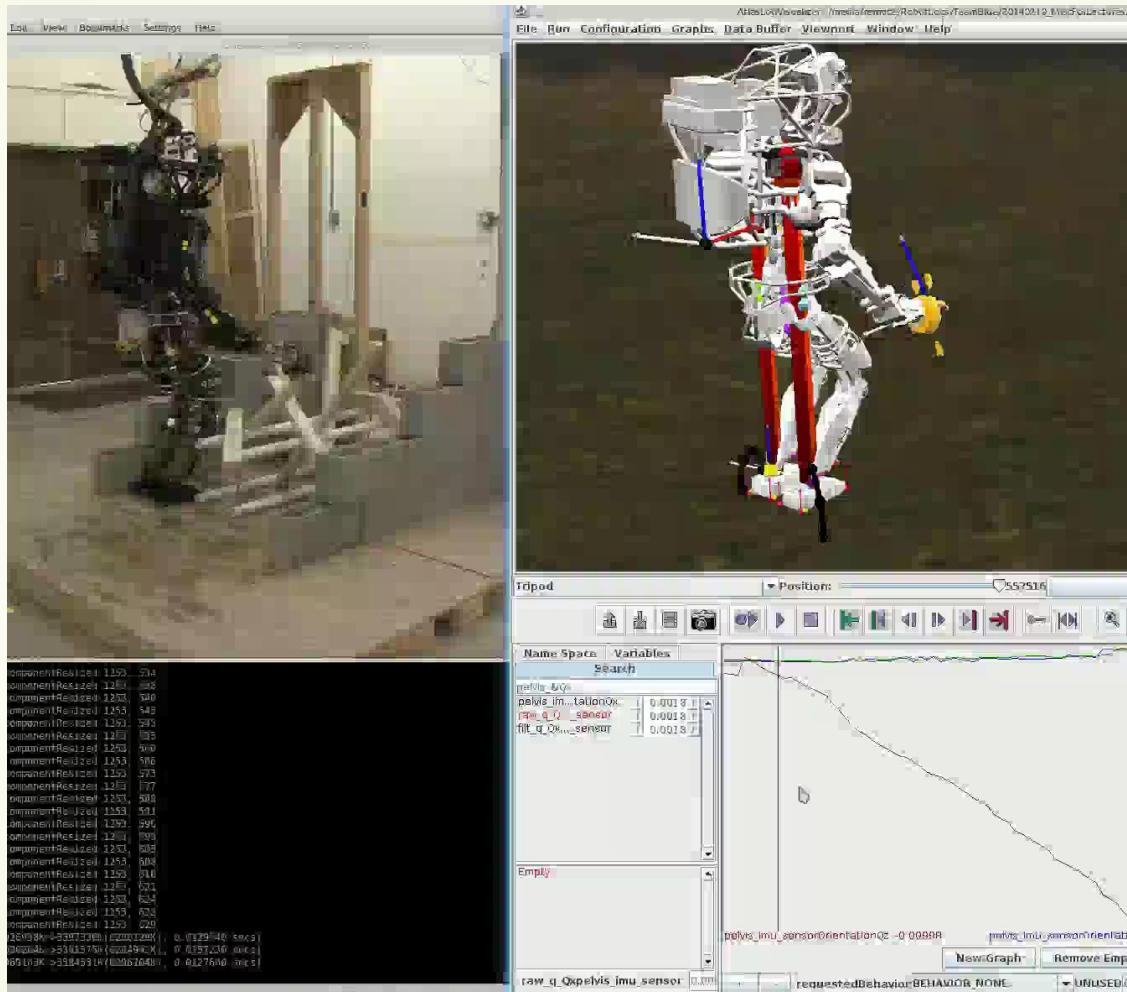
Issues: Foot slipping



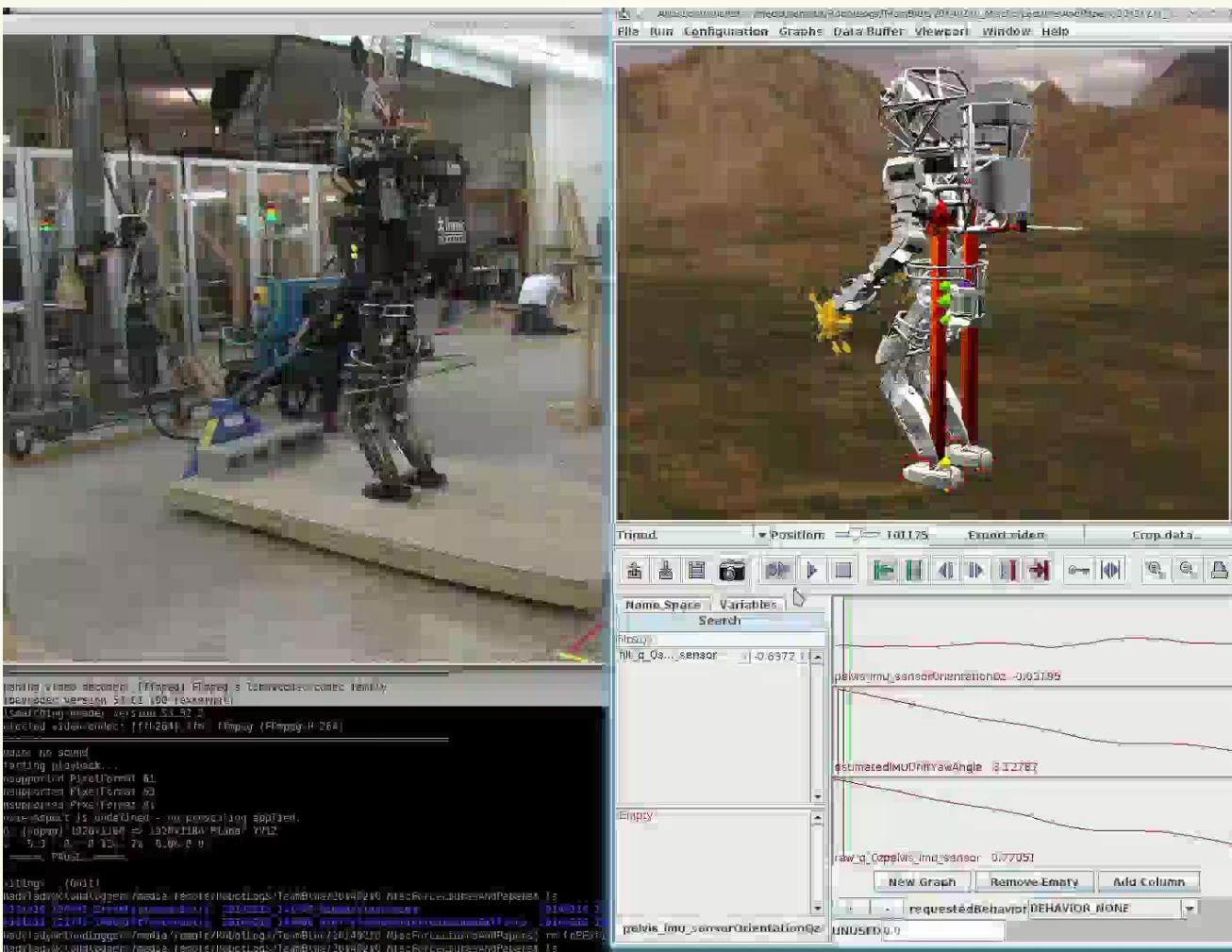
Foot slipping

A dark, low-light photograph showing a close-up of a robotic leg in motion. The leg appears to be made of metal and plastic, with a blue light source highlighting its structure. The background is mostly black, suggesting a dark environment or a low-light setting.

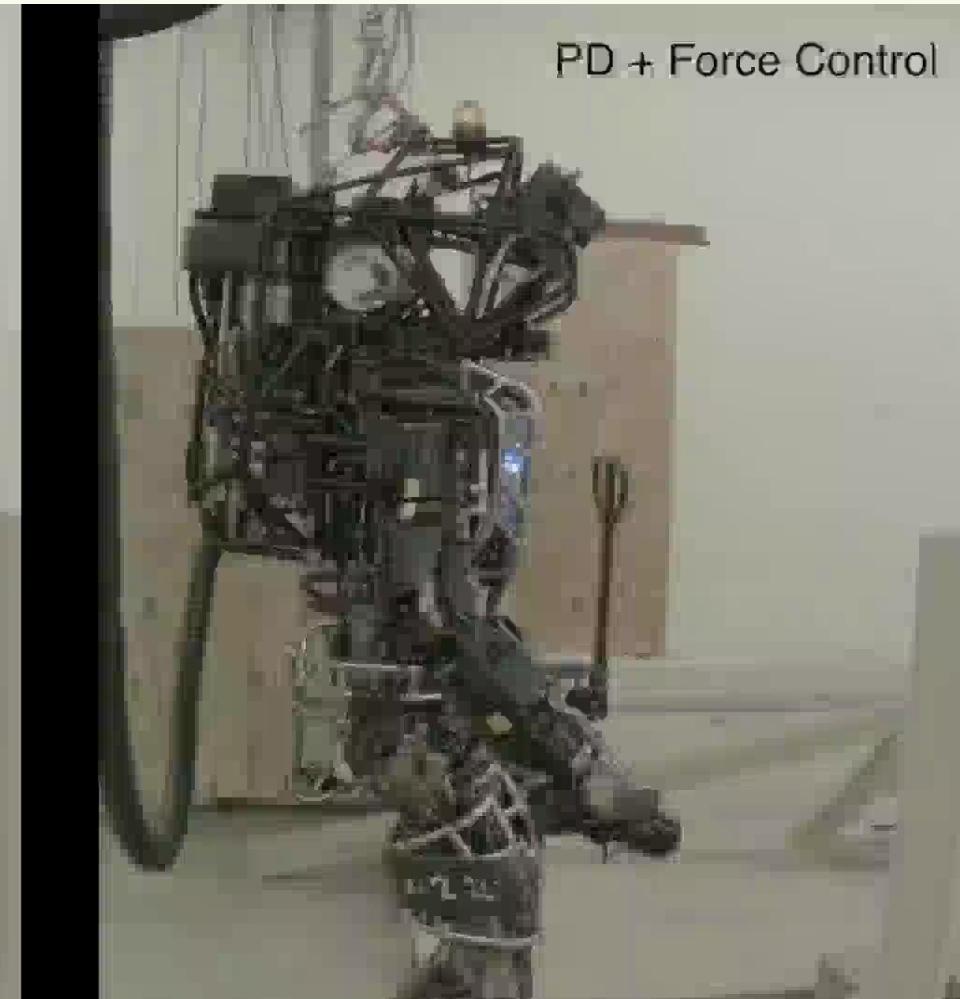
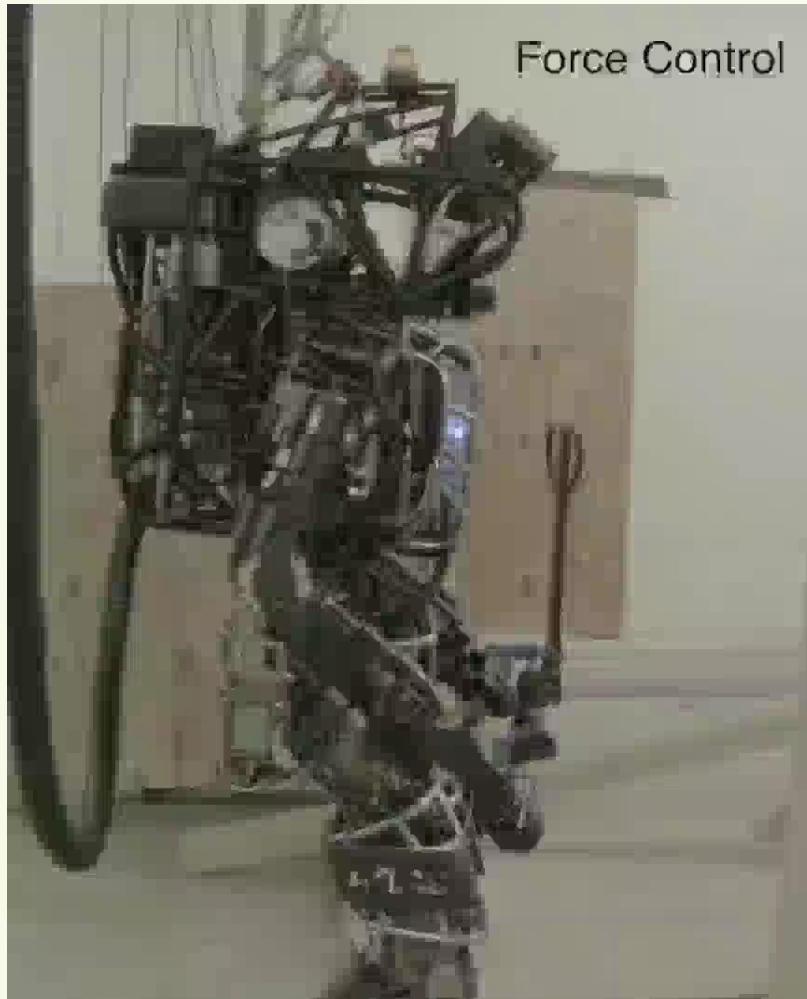
Issues: IMU drift



Issues: IMU drift

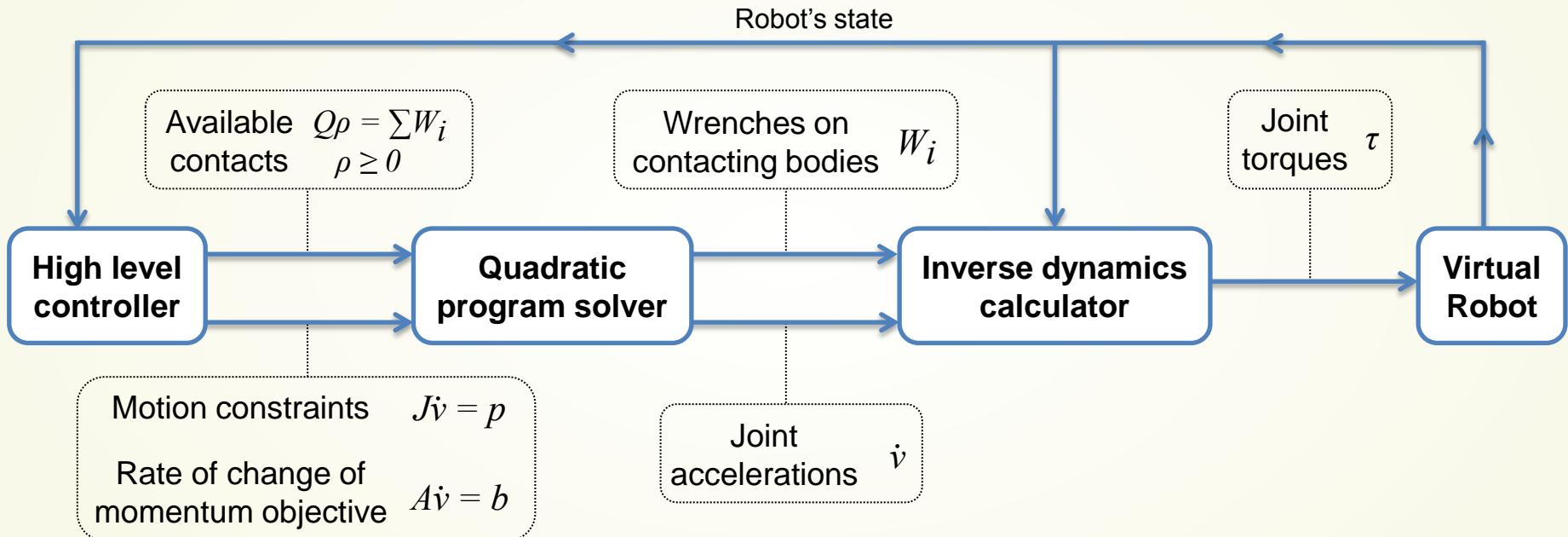


Issues: Joint stiction



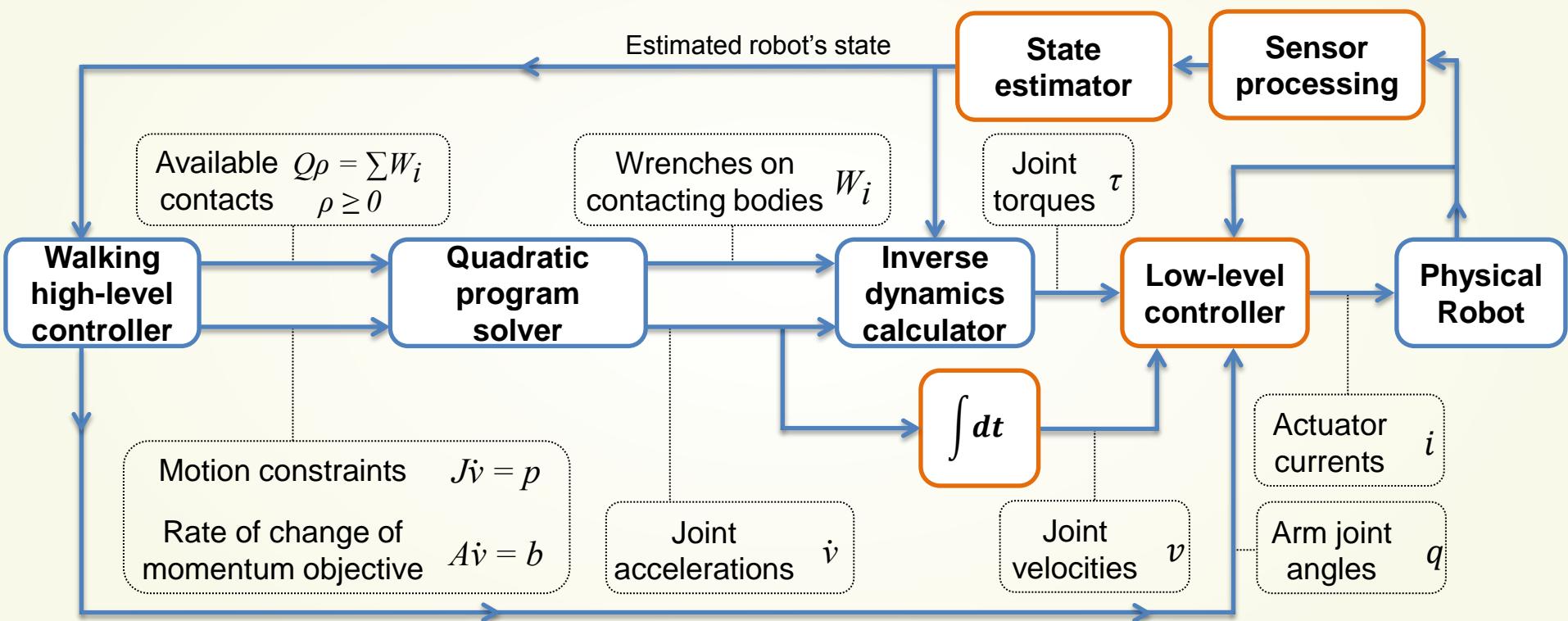
Whole-body motion control framework

Before



Whole-body motion control framework

After



Sensor processing

- Sensor noise: low-pass filters
- Link elasticity:

$$q_{proc} = q_{raw} + \frac{\tau}{k}$$

- Joint backlash:

$$\dot{q}_{proc}^t = \alpha \dot{q}_{proc}^{t-1} + (1 - \alpha) \dot{q}_{raw} \eta$$

$$\text{with: } \eta = \begin{cases} \frac{t-t_0}{\Delta t} & \text{if } (t - t_0) < \Delta t \\ 1 & \text{else} \end{cases}$$

$t_0 = t$ when \dot{q}_{proc} crosses zero

State Estimator (IMU drift)

- Built new state estimate for Atlas:
 - Trusts IMU for orientation and angular velocity
 - Fuses data for position and linear velocity:
$$\dot{x}^t = \alpha_{\dot{x}}(\dot{x}^{t-1} + \ddot{x}_{IMU}^t \Delta t) + (1 - \alpha_{\dot{x}})\dot{x}_{kin}$$
$$x^t = \alpha_x(x^{t-1} + \dot{x}^t \Delta t) + (1 - \alpha_x)x_{kin}$$
- Compensator for IMU drift in orientation and angular velocity
- State estimator error:
 - $\sim 1\text{inch/step}$ on the horizontal position
 - $\sim 0.2\text{inch/step}$ along the vertical

Walking controller (chatter/shakies)

- Chatter:
 - Reduced PD controller gains (especially damping)
 - Motion constraints: limited to a maximum acceleration and maximum jerk
- Model inaccuracies:
 - System ID (for the chest mass)
 - Added integral term to the ICP controller:

$$r_{CMP}^{des} = r_{ICP} + \frac{K_p}{\omega_0} (r_{ICP}^{des} - r_{ICP}) + \frac{K_i}{\omega_0} \int (r_{ICP}^{des} - r_{ICP}) dt - \frac{1}{\omega_0} \dot{r}_{ICP}^{des}$$

Low-level controller (joint stiction)

- Hybrid controller to fight stiction:

$$i = k_{ff_{qd}} \dot{q} + i_\tau + i_{\dot{q}} + i_q$$

with:
$$\begin{cases} i_\tau = k_{\tau,p}(\tau_d - \tau) + k_{\tau,d}(\dot{\tau}_d - \dot{\tau}) + k_{\tau,i} \int (\tau_d - \tau) dt \\ i_{\dot{q}} = k_{\dot{q}} \left(\int \ddot{q}_d dt - \dot{q} \right) \\ i_q = k_{q,p}(q_d - q) + k_{q,v}(\dot{q}_d - \dot{q}) + k_{q,i} \int (q_d - q) dt \end{cases}$$

- i_τ : PID controller on joint torque
- $i_{\dot{q}}$: P controller on joint velocity
- i_q : PID controller on joint position
- Arm joints: torque + position controller ($i = k_{ff_{qd}} \dot{q} + i_\tau + i_q$)
- Other joints: torque + velocity controller ($i = k_{ff_{qd}} \dot{q} + i_\tau + i_{\dot{q}}$)
 - Joint desired velocity obtained from joint desired acceleration (QP solver output)
 - Support ankle joints: only torque controller ($i = k_{ff_{qd}} \dot{q} + i_\tau$)

Good runs

Cool beans

At the end

- Control
 - Wanted a compliant control ended up with high impedance
 - Introduced lots of “hacks” that needs to be cleaned and improved
- Atlas
 - Gave us hard time
 - Pretty repeatable fitting our testing obsession

Future work

- General improvements
 - Modeling
 - State estimation
 - Low-level controller
 - Push-recovery
 - Fast walking

Thank you!

A big thanks to everyone who contributed:

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