Balancing Experiments on a Torque Controlled Humanoid with Hierarchical Inverse Dynamics

Max-Planck Institute Intelligent Systems (Autonomous Motion)

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Low-level control of Sarcos humanoid

- Lower body of Sarcos humanoid
- 17 DOFs
- Linear hydraulic actuators
- Moog 30 Series valves
- Load cells and position sensors at each joint
- 6-axis force sensors in the foot
- 1KHz control loop

Control of hydraulics

- How do we implement a "torque source" with hydraulics?

[Boaventura et al. 2012]

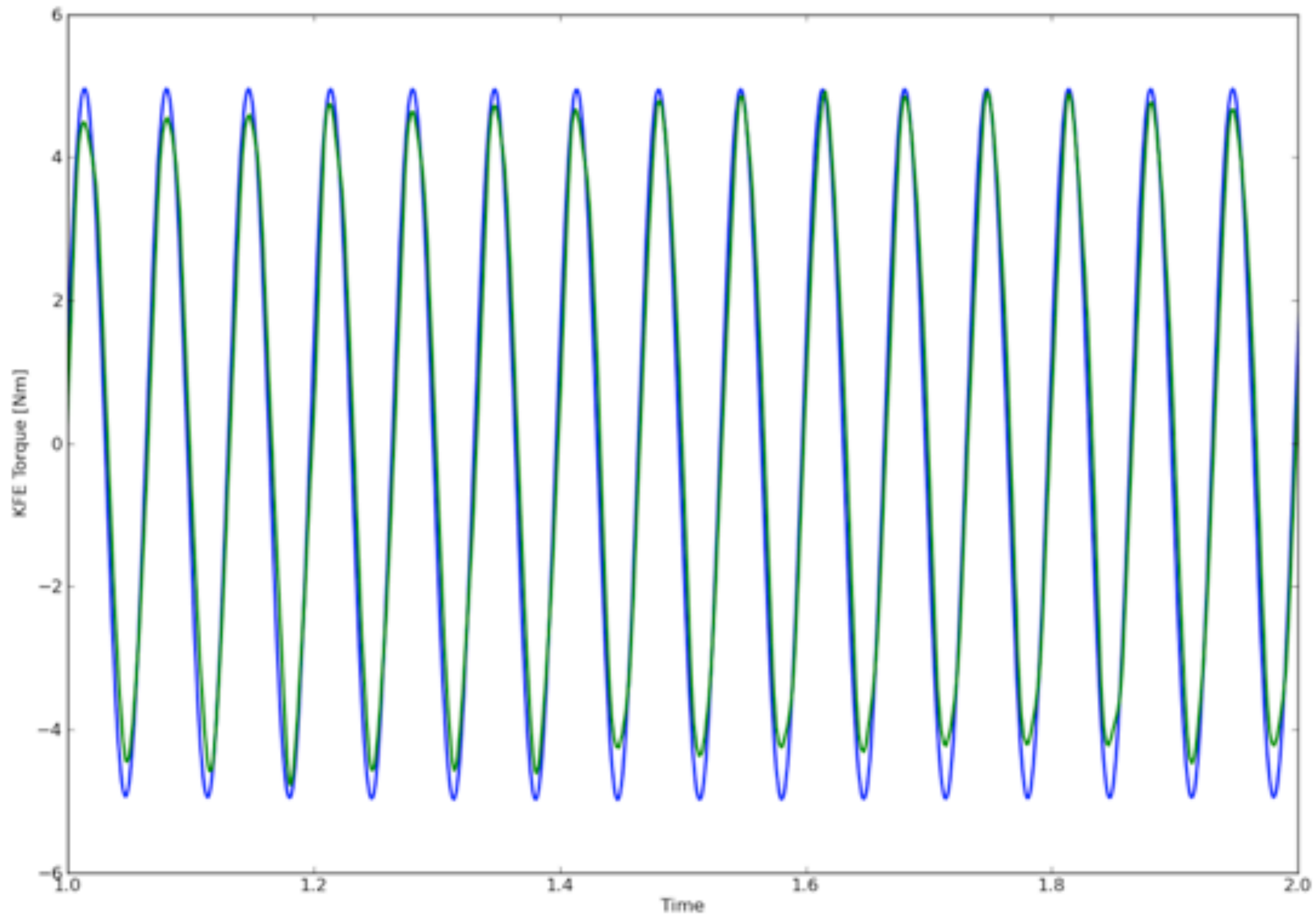
$$v = PID(F_{des}, F) + K\dot{x}_{piston} + c$$

- This controller was key to good force control performances

- Special care in calibration and tuning of controller to maximize performance

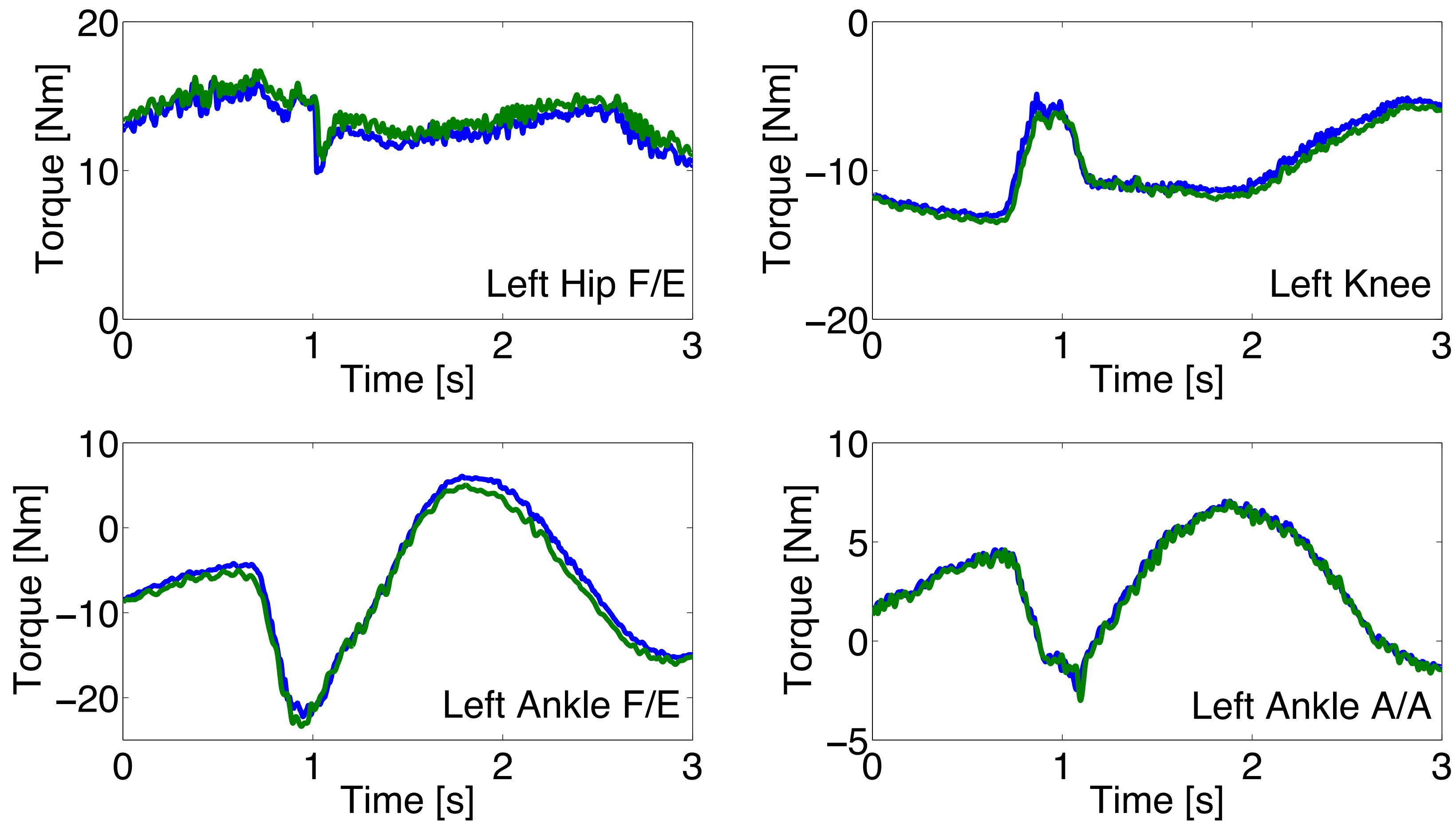
Torque tracking performance

[Boaventura et al. 2012]



Tracking a 15Hz sine torque profile (5Nm amplitude)

Torque tracking performance



Control of hydraulics

- Painful to tune controllers for each DOF
- Control performance not so good for ankles
  (velocity compensation gain really depends on position)
- Now: looking into automatic tuning / learning control
  (current work by S. Trimpe)

Inverse dynamics for legged robots

[Righetti et al., IJRR 2013]

QR decomposition of constraint Jacobian
[Mistry et al. 2010]

$$\mathbf{J}_c^T = [\mathbf{Q}_c \ \mathbf{Q}_u] \left[\begin{array}{c} \mathbf{R} \\ \mathbf{0} \end{array} \right]$$

Equations of motion: $\mathbf{M}\ddot{\mathbf{q}} + \mathbf{h} = \mathbf{S}^T \boldsymbol{\tau} + \mathbf{J}_c^T \boldsymbol{\lambda}$

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Equations of motion:

$$\mathbf{Q}_u^T(\mathbf{M}\ddot{\mathbf{q}}_d + \mathbf{h}) = \mathbf{Q}_u^T\mathbf{S}^T\boldsymbol{\tau}$$

Dynamic consistency

$$\boldsymbol{\lambda} = \mathbf{R}^{-1}\mathbf{Q}_c^T(\mathbf{M}\ddot{\mathbf{q}}_d + \mathbf{h} - \mathbf{S}^T\boldsymbol{\tau})$$

Contact force as a function of actuation

Optimal distribution of contacts

[Righetti et al. 2012]

$$\arg\min_{\boldsymbol{\tau},\boldsymbol{\lambda}} \frac{1}{2}\boldsymbol{\tau}^T \mathbf{W}_\tau \boldsymbol{\tau} + \mathbf{b}_\tau^T \boldsymbol{\tau} + \frac{1}{2}\boldsymbol{\lambda}^T \mathbf{W}_\lambda \boldsymbol{\lambda} + \mathbf{b}_\lambda^T \boldsymbol{\lambda}$$

$$s.t. \quad \mathbf{Q}_u^T \mathbf{S}^T \boldsymbol{\tau} = \mathbf{Q}_u^T (\mathbf{M}\ddot{\mathbf{q}}_d + \mathbf{h})$$ Dynamic consistency

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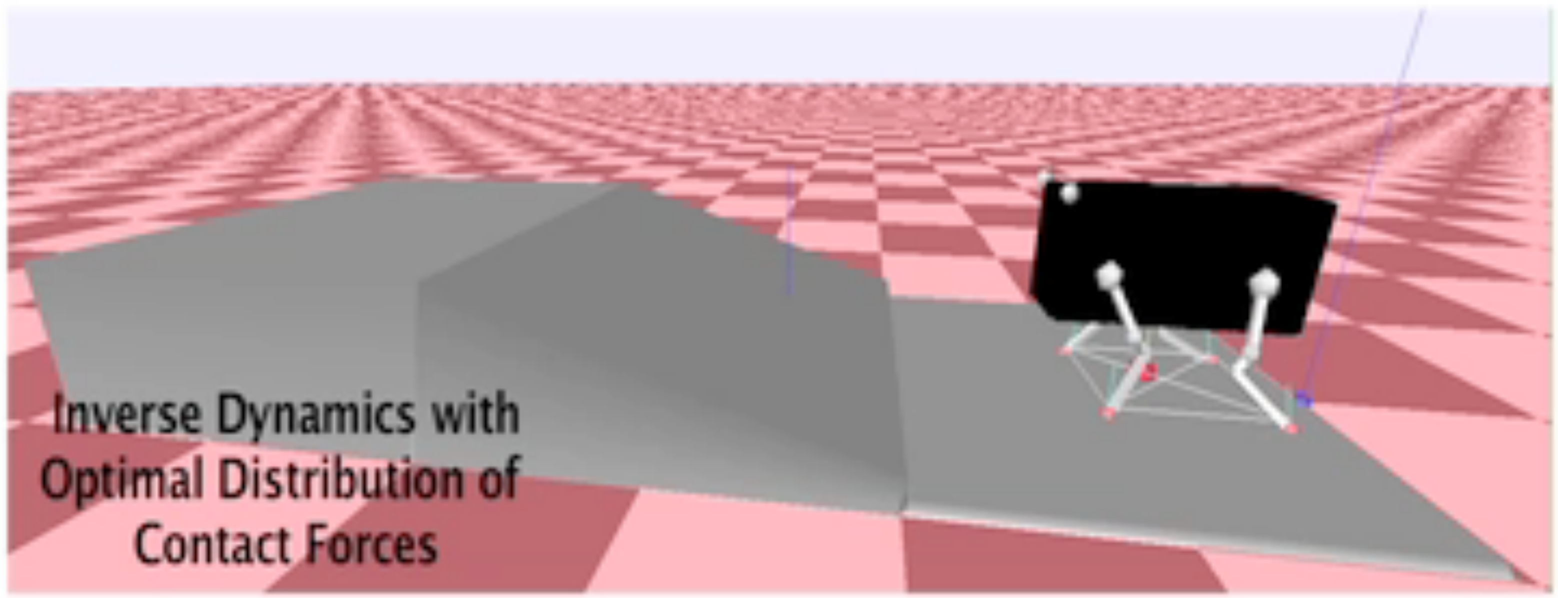
Dynamic consistency

Torque constraints

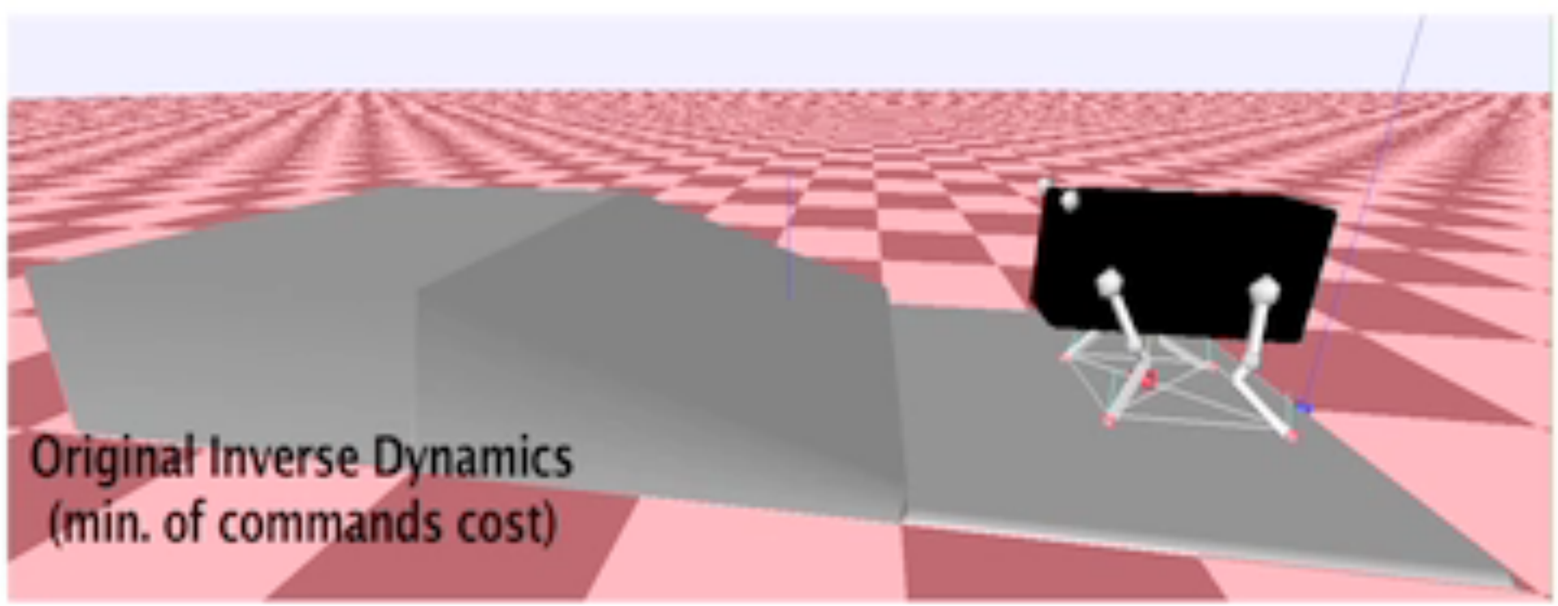
Contact forces constraints

- QP depends only on torques
- No need of an explicit representation of contact forces
- Problem with constant size - no matter the number of contacts

Optimal distribution of contact forces



Climbing a 0.25 radians slope with low friction (coefficient of static friction = 0.4)



Inverse dynamics for legged robots

[Righetti et al., IJRR 2013]
[Mistry et al., 2010]

- Inverse dynamics (joint acceleration) and operational space (task space accelerations) control
- Torque redundancy to optimize contact forces
- Computationally fast (1KHz control loop)
- Robust to model uncertainties / better with system identification (no need to compute inertia matrix)

Related Work

- Passivity-based: exploit quasi-static assumption [Hyon et al, 2007][Ott et al, 2011]

  + robustness due to passivity; no need for precise dynamics model

  - assumptions are potentially limiting for dynamic motions

- Control with full Dynamical Model [Stephens et al, 2010][Hutter et al, 2012][Righetti et al, 2013]

  + theoretically well suited for dynamic motions

  - requires model and efficient implementation

Motivation

define desired closed-loop dynamics

e.g. $\mathbf{J_x}\ddot{\mathbf{q}} = PD(\mathbf{x}_{des}, \dot{\mathbf{x}}_{des})$ (1)

- Typical use of the presented framework:
  - define desired closed-loop dynamics
  - pick one $\ddot{\mathrm{q}}*$ (out of many) that satisfies Eq (1)

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exploit redundancy to optimize cost on torques or forces

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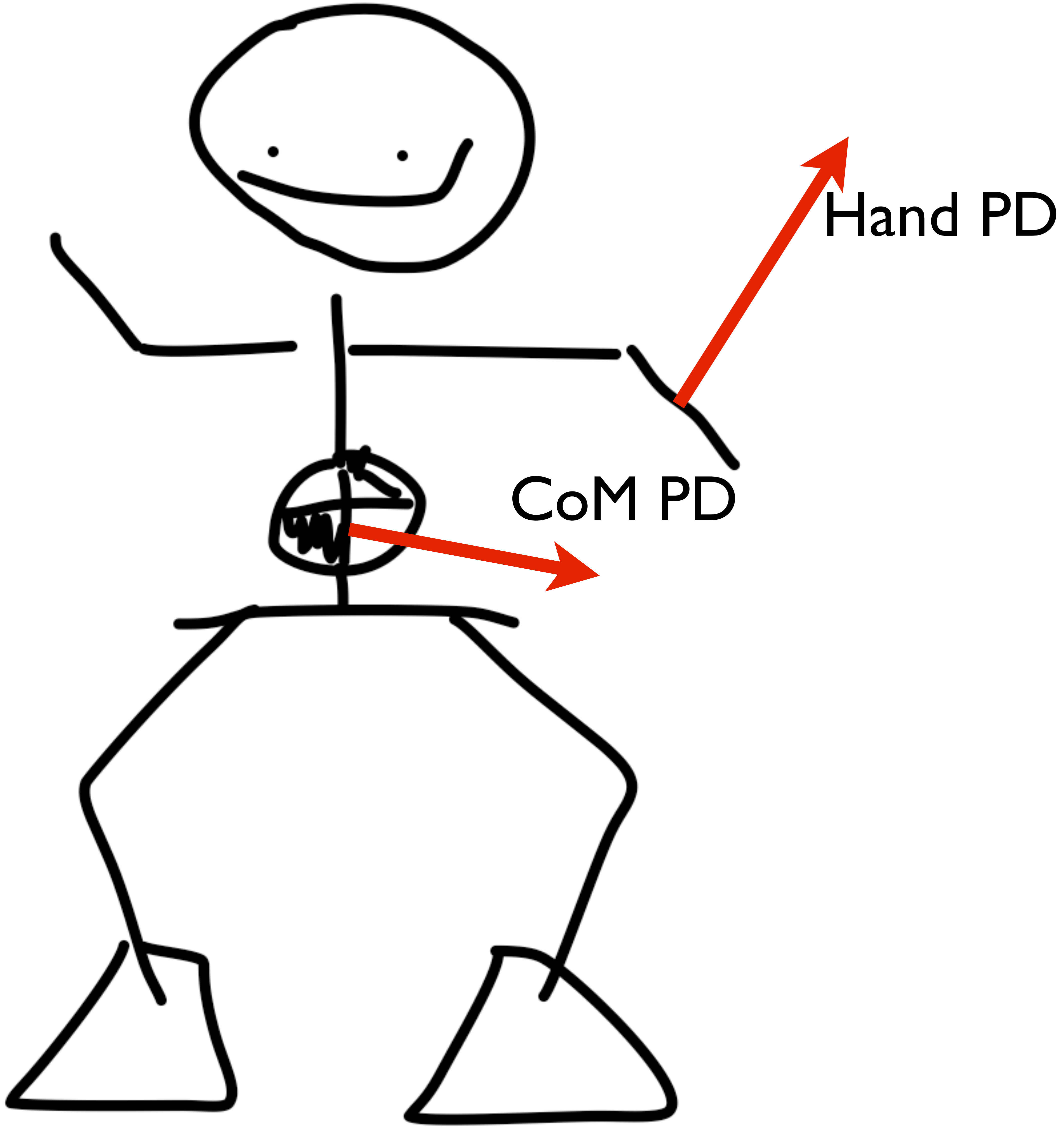
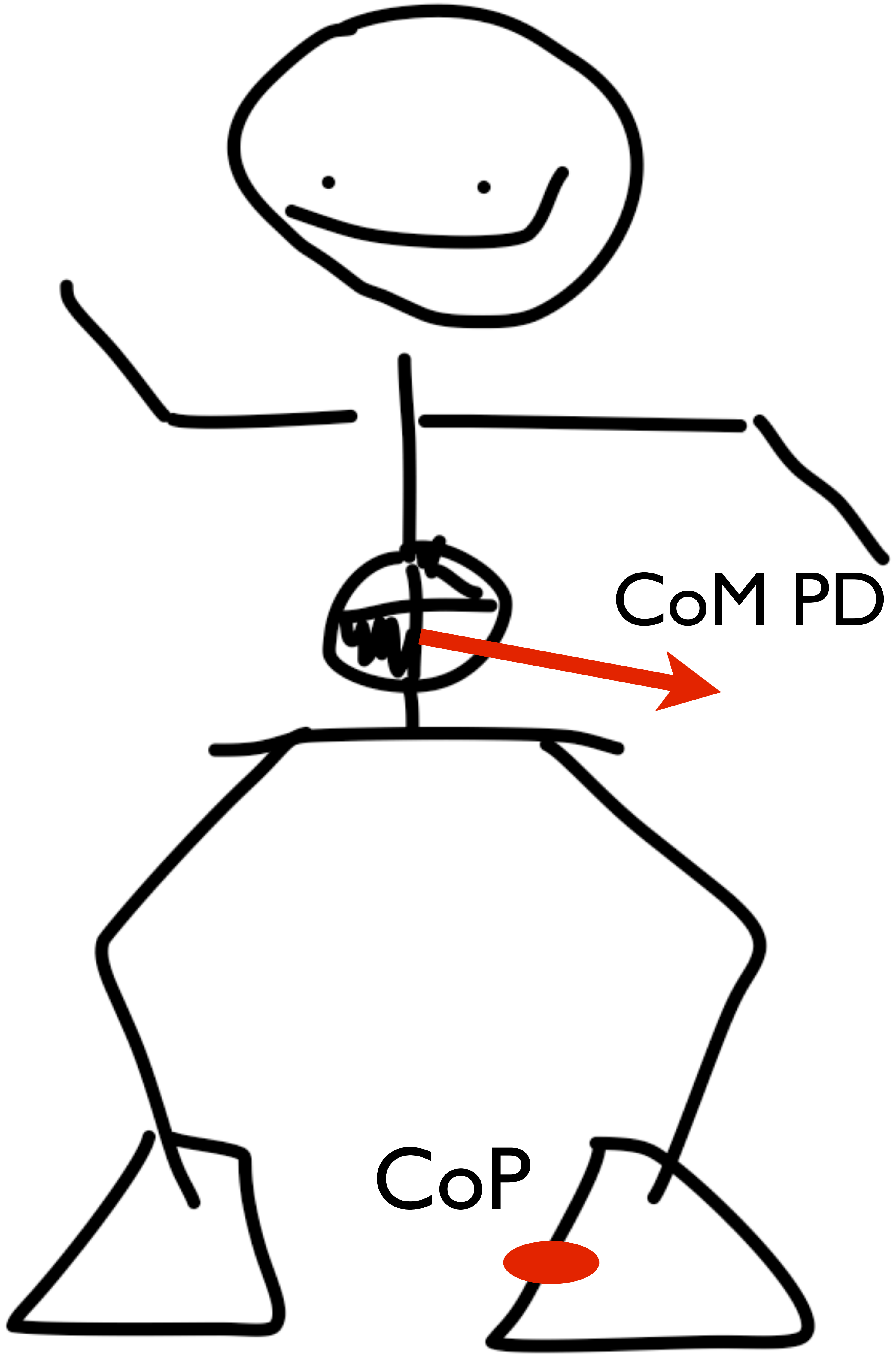
- Typical use of the presented framework:
  - define desired closed-loop dynamics
  - pick one $\ddot{q}^*$ (out of many) that satisfies Eq (1)
  - optimize over space of redundant torques and forces
- Potentially suboptimal or even infeasible by ignoring all other solutions to Eq (1)
- In addition it is useful to be able to express hierarchies on inequalities

Hierarchies



- We want
  - the CoM to have PD behavior
  - the hand as well
- what if both cannot be satisfied?
- => weighting might help

Hierarchies



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- We want
  - the CoM to have PD behavior
  - CoP to reside inside support polygon
- if CoP constr. violated => kinematics constraint wrong
- => hierarchies

Related Work

- cascades of QPs: recursively solve a QP without violating optimality of previous QPs [de Lasa,2010, Mansard, 2012]
    - generalize pseudo-inverse approaches: allow inequality constraints
    - have not been implemented in a feedback-loop on a robot before
    - requires efficient implementation to run on torque controlled robot
    - how well does it perform under model-uncertainty, noisy velocity measures and realistic base-state estimation?

Cascades of QPs

[deLasa et. al., 2010]

$$\mathbf{y} = \begin{bmatrix} \ddot{\mathrm{q}} \\ \boldsymbol{\tau} \\ \boldsymbol{\lambda} \end{bmatrix}$$

$$\mathbf{W}_1(\mathbf{B}_1\mathbf{y} + \mathbf{b}_1) = \mathbf{w}_1$$

Cascades of QPs

[deLasa et. al., 2010]

QP 1: $$\min_{\mathbf{y},\mathbf{v}_1,\mathbf{w}_1} \ \|\mathbf{v}_1\|^2 + \|\mathbf{w}_1\|^2$$

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All optimal solutions:

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$\mathbf{Z}_i$ surjective map onto $\bigcap_{j=1}^{i} \text{Nullspace}(\mathbf{W}_j\mathbf{B}_j)$

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→ ...

Computation Time

- Solving a cascade of QPs in 1ms requires an efficient implementation
- QP variables: n+6 + n + 6*c (c = number of constrained endeffectors)
- Highest priority objective: $$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau} + \mathbf{J}_c^T\boldsymbol{\lambda}$$ $$\tilde{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}} + \tilde{\mathbf{N}}(\mathbf{q},\dot{\mathbf{q}}) = \tilde{\mathbf{J}}_c^T\boldsymbol{\lambda}$$
- By substituting $\boldsymbol{\tau} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{J}_c^T\boldsymbol{\lambda}$ we save n variables and n constraints!

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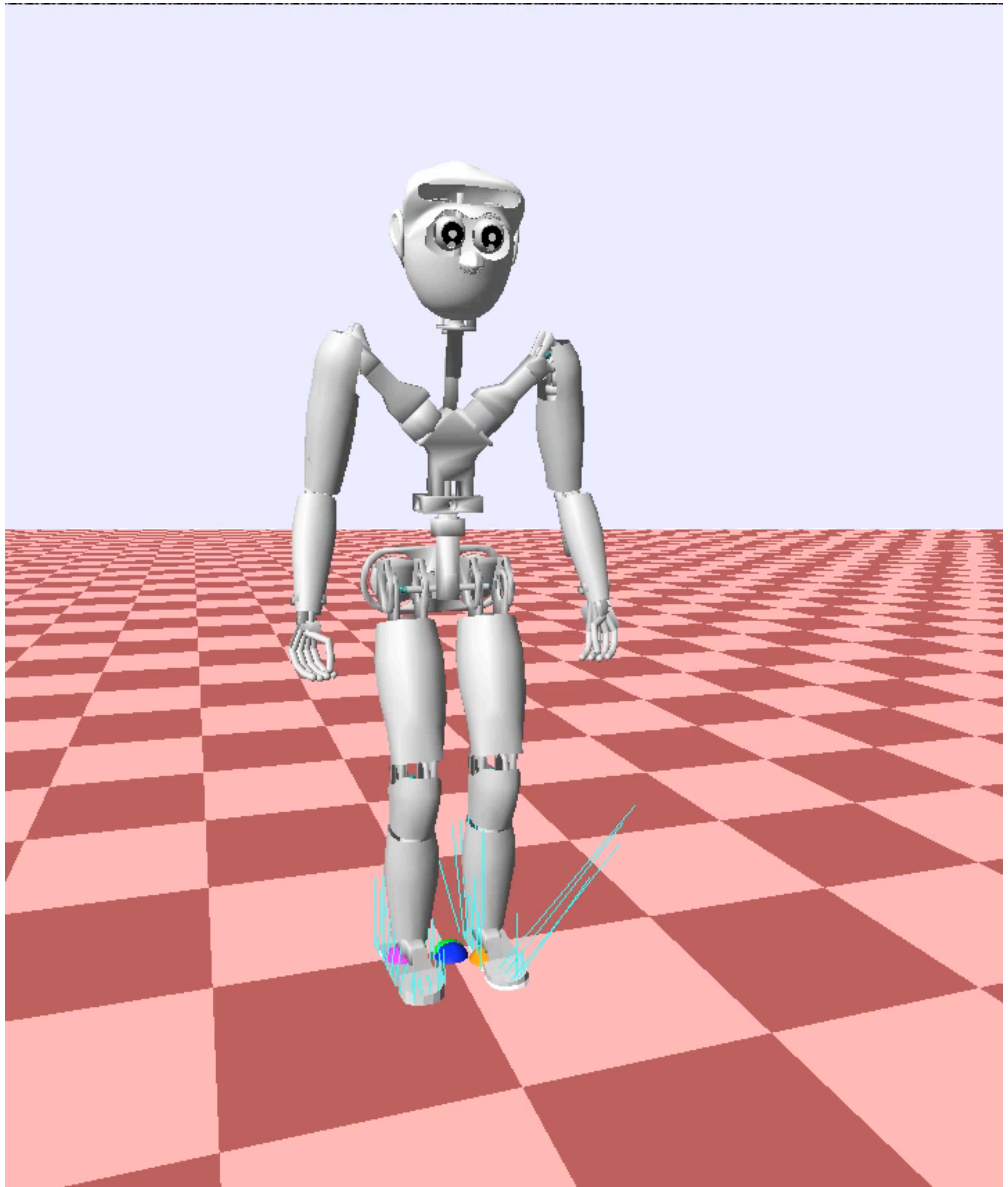
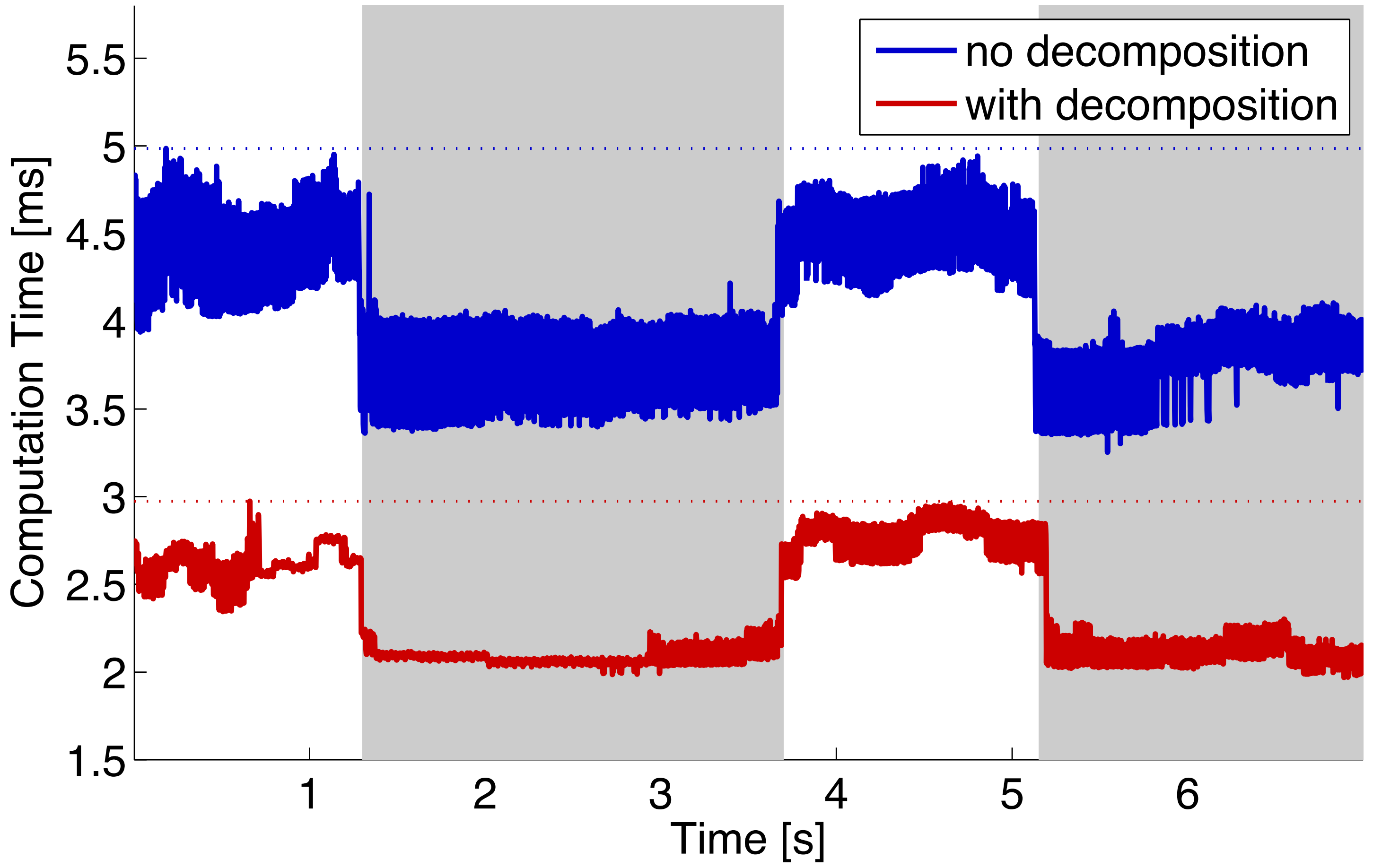
- first hierarchie is always EoM and torque constraints => no QP needs to be solved

Speedup



Priority	Nr. of eq(uality) and ineq(uality) constraints	Constraint/Task
1	25 eq	Eq. (12) (not required for simplified problem)
	6 eq	Newton Euler Eq. (13)
	2×25 ineq	torque limits
2	$c \times 6$ eq	kinematic contact constraint
	$c \times 4$ ineq	CoPs reside in sup. polygons
	$c \times 4$ ineq	GRFs reside in friction cones
	2×25 ineq	joint acceleration limits
3	3 eq	PD control on CoM
	$(2 - c) \times 6$	PD control on swing foot
4	$25 + 6$ eq	PD control on posture
5	$c \times 6$ eq	regularizer on GRFs
	DoFs: 25	**max. time:** 5 ms / 3 ms



Momentum Control

$$\mathbf{H_G}(\mathbf{q})\dot{\mathbf{q}} = \mathbf{m}$$

[Orin, & Goswami 2008]

$$\dot{\mathbf{m}}_{ref} = \mathbf{P}\begin{bmatrix} M(\mathbf{x}_{cog,des} - \mathbf{x}_{cog}) \\ \mathbf{0} \end{bmatrix} + \mathbf{D}(\mathbf{m}_{des} - \mathbf{m}) + \dot{\mathbf{m}}_{des}$$

$$\dot{\mathbf{m}} = \mathbf{H_G}\ddot{\mathbf{q}} + \dot{\mathbf{H}}_\mathbf{G}\dot{\mathbf{q}}$$
$$= \begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{0}_{3\times3} & \dots \\ [\mathbf{x}_{cog} - \mathbf{x}_i]_\times & \mathbf{I}_{3\times3} & \dots \end{bmatrix} \boldsymbol{\lambda} + \begin{bmatrix} M\mathbf{g} \\ \mathbf{0} \end{bmatrix}$$

No meaningful integral of angular momentum (orientation)

choice of ang. mom. for motions is non-intuitive. putting it in nullspace of motion generates undesirable behavior

requires deriving H numerically => can suffer from noise

Momentum-based Balance Control

Priority	Nr. of eq(uality) and ineq(uality) constraints	Constraint/Task
1	6 eq	Newton Euler Eq. (13)
	2×14 ineq	torque limits
	2×6 eq	kinematic contact constraint
	2×4 ineq	CoPs reside in sup. polygons
	2×4 ineq	GRFs reside in friction cones
	2×14 ineq	joint acceleration limits
2	6 eq	PD control on system momentum, Eq. (17)
	$14 + 6$ eq	PD control on posture
	2×6 eq	regularizer on GRFs
	DoFs: 14	**max. time:** 0.4 ms

- formulation requires only one QP
- runs solidly below 1ms
- guarantees dynamic constraints; makes no trade-offs

Balancing experiments on a torque-controlled humanoid with hierarchical inverse dynamics

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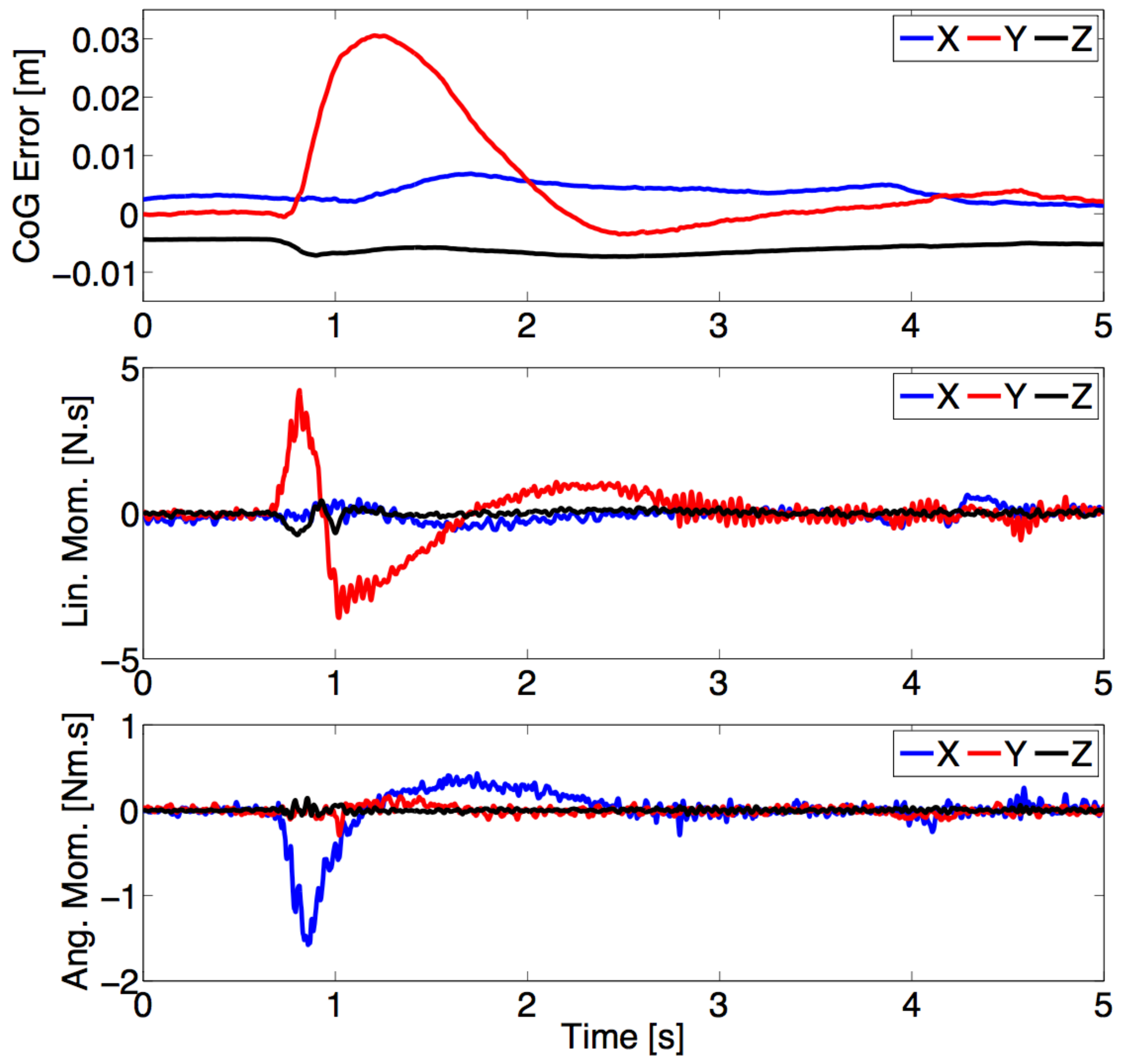
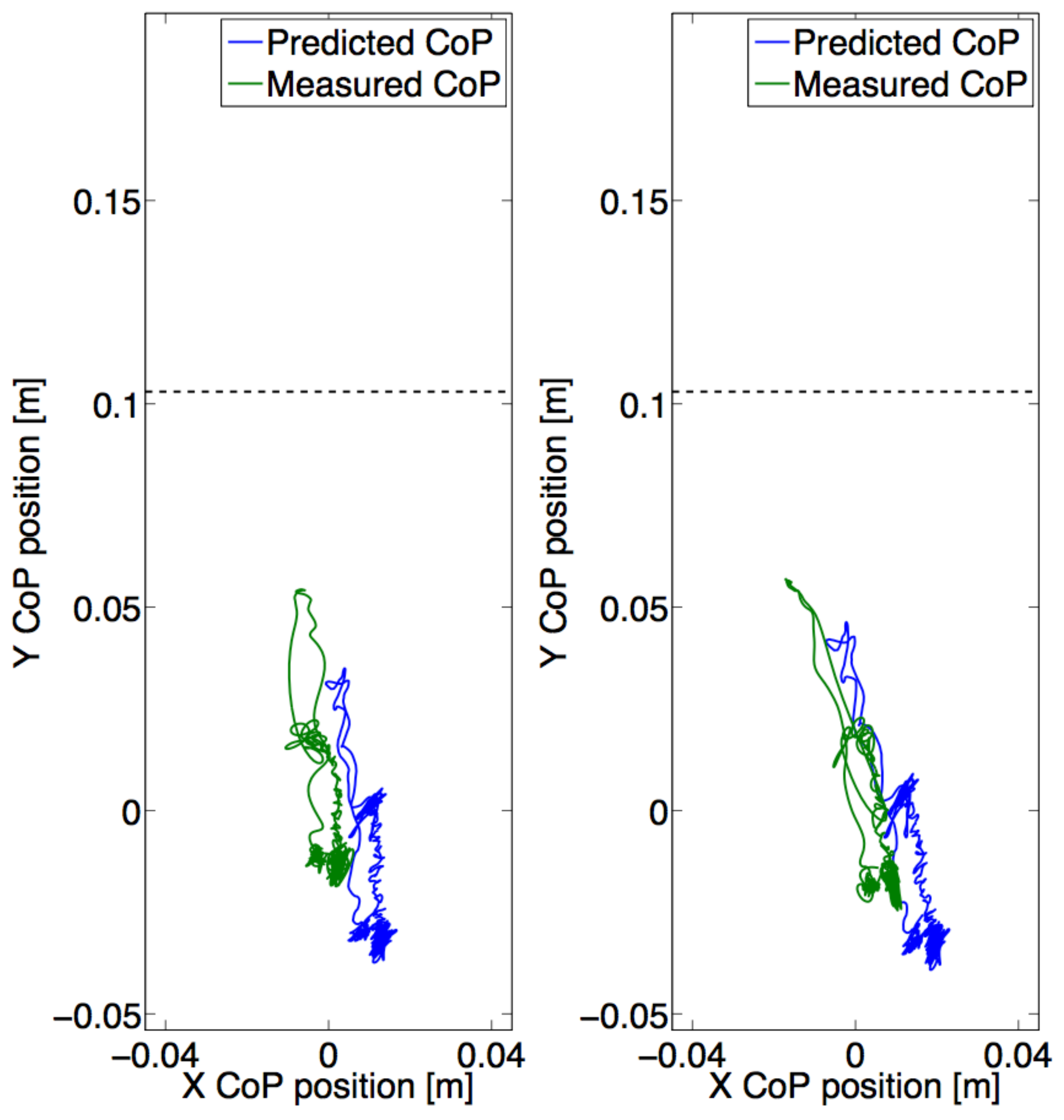
Momentum-based Balance Control





- Recovers CoM position after push
- guarantees admissible CoPs and predicts these reliably

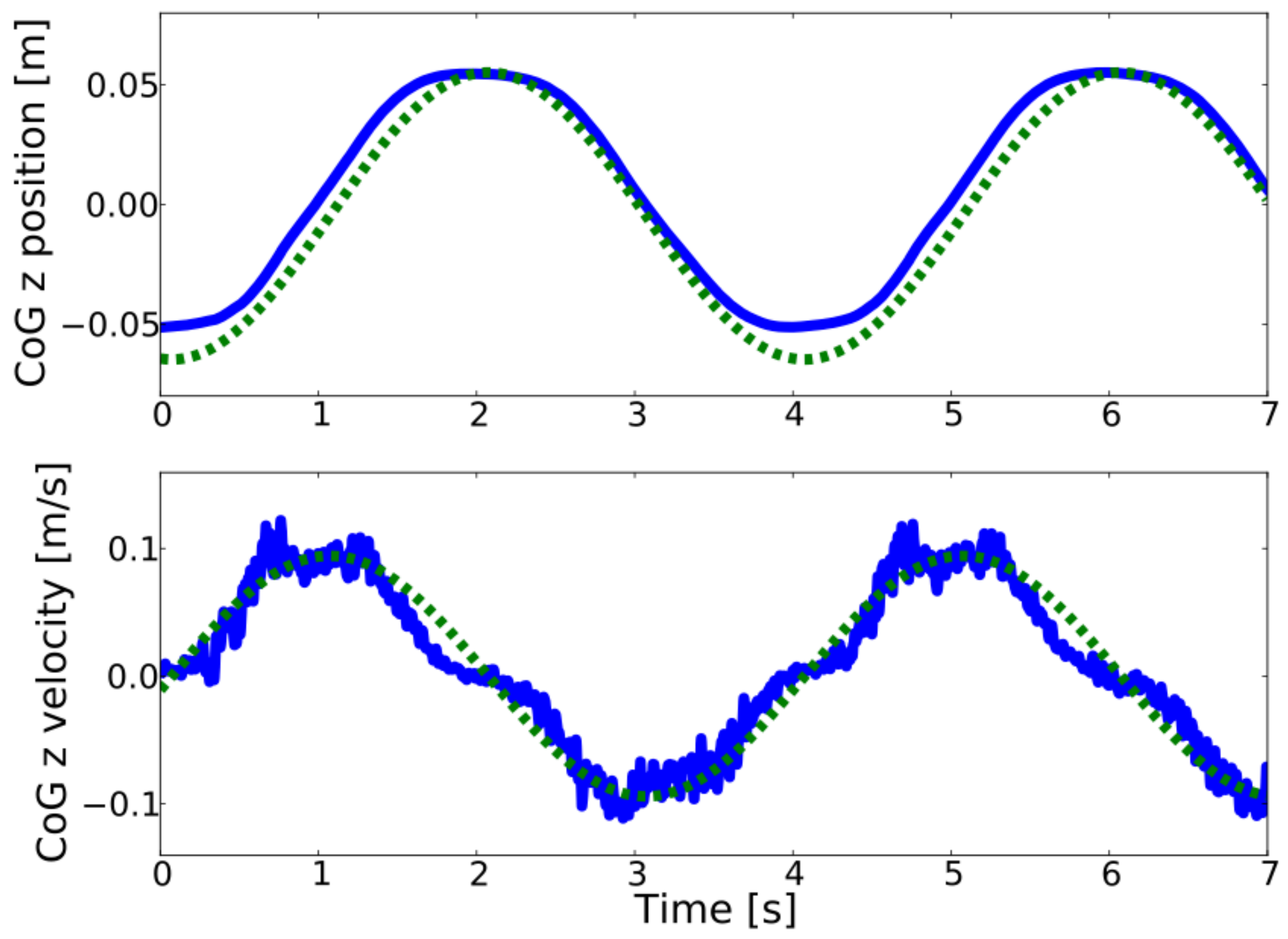
Squatting

Priority	Nr. of eq(uality) and ineq(uality) constraints	Constraint/Task
1	6 eq	Newton Euler Eq. (13)
	2×14 ineq	torque limits
2	2×6 eq	kinematic contact constraint
	2×4 ineq	CoPs reside in sup. polygons
	2×4 ineq	GRFs reside in friction cones
	2×14 ineq	joint acceleration limits
3	3 eq	PD control on CoG
4	$14 + 6$ eq	PD control on posture
5	2×6 eq	regularizer on GRFs
	DoFs: 14	**max. time:** 0.9 ms

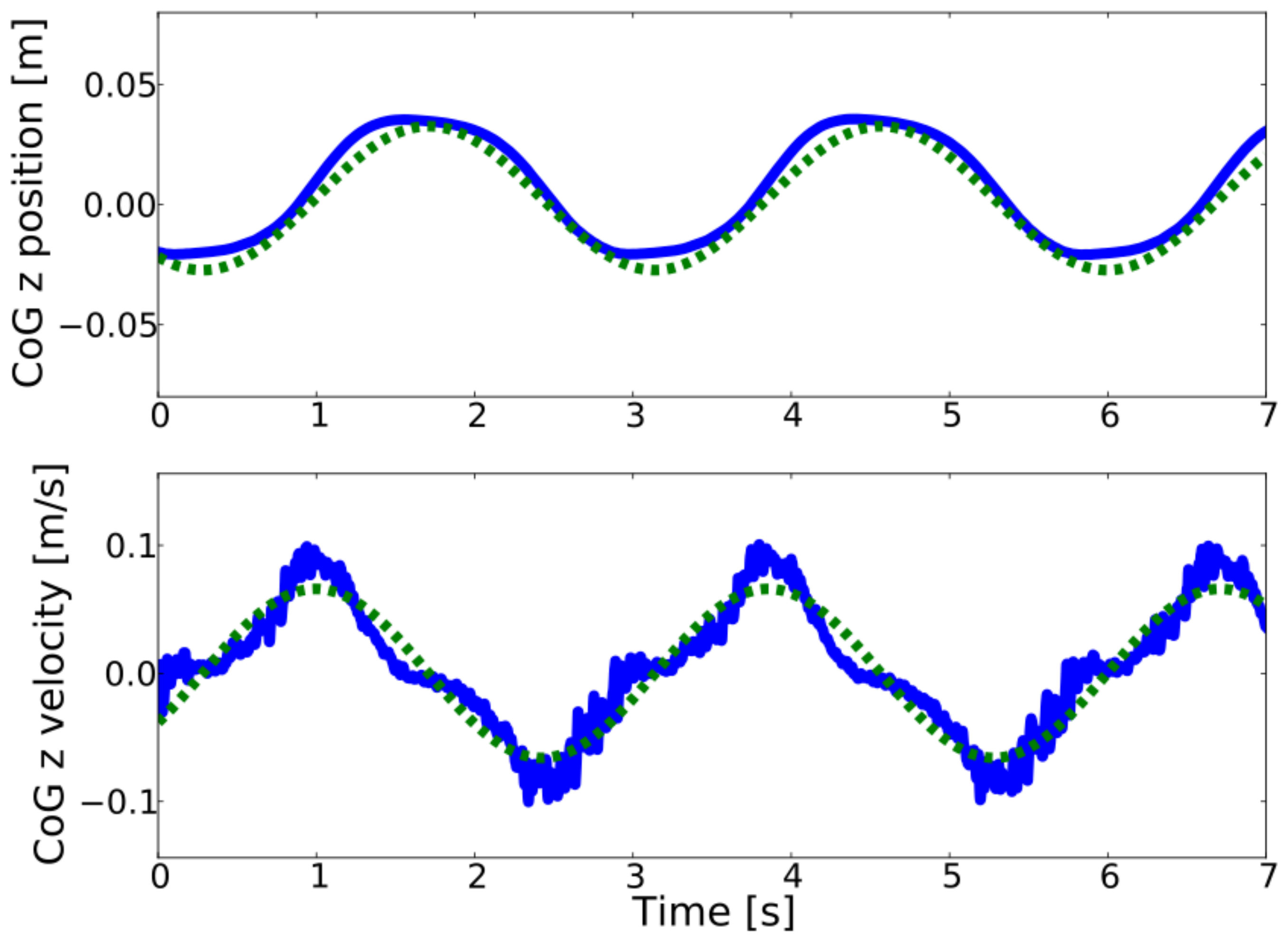
- prioritize

1. dynamic constraints
2. CoM motion tracking
3. redundancy resolution on motion and forces

Squatting



(a) 0.25 Hz high amplitude tracking task



(b) 0.3 Hz low amplitude tracking task

- We can track CoM tasks of different frequencies
- Posture and GRFs are optimized in a lower hierarchy
- allows for balancing up to some extend in face of disturbances
- no ang. mom. control

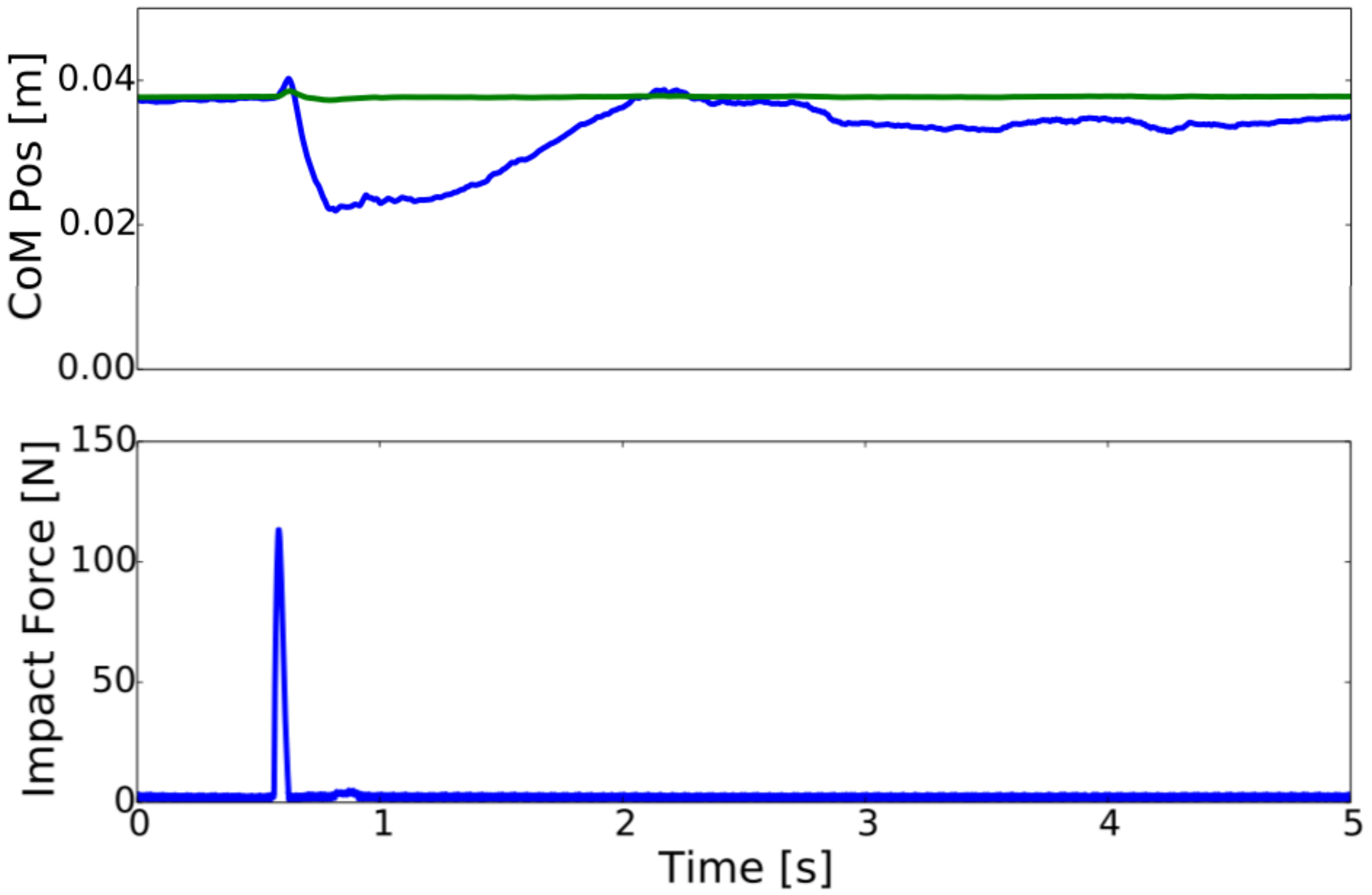
Balance in Single-Support

Priority	Nr. of eq(uality) and ineq(uality) constraints	Constraint/Task
1	6 eq	Newton Euler Eq. (13)
	2×14 ineq	torque limits
2	2×4 ineq	CoPs reside in sup. polygons
	2×4 ineq	GRFs reside in friction cones
	2×14 ineq	joint acceleration limits
3	6 eq.	Linear and angular momentum control
	12/6 eq.	kinematic contact constraint
	0/6 eq.	Cartesian foot motion (swing)
	14 eq.	PD control on posture
4	$2 \times 6/1 \times 6$ eq.	regularizer on GRFs
	DoFs: 14	**max. time:** 1.05 ms

- moving on one leg requires contact switches (problematic in hierarchies)
- We put kinematic contact constraints and swing foot task into the same hierarchy to avoid the problem


Contact switching and
Balancing on one foot

Balance in Single-Support



- F/T Sensor attached to stick
- Impulse comparably high to related work* (4.5 to 5.8 Ns)
- transitioning phase for contact switch

*[Ott, 2011]

Notes on Experiments

- We feed forward torques from solver directly. No additional joint control
- Limiting Factors:
  - Naive state estimation
  - dynamic model is obtained from CAD
  - Lag of feasible trajectories => Slacks due to inconsistency with EoM => closed-loop dynamics not achieved

Notes on Experiments

- Solver Formulation:
  - slacks help analyzing conflicts in control
  - the more hierarchies the less intuitive the behavior
  - unclear what des ang. mom. should be when moving
- theoretically QP cascades generate smooth trajectories, when problem changes smoothly, but the slopes can be very high if many inequality constraints become active

- bad velocity readings
- so far no really dynamic motions => are quasi-static approaches sufficient?

Conclusion

- cascades of QPs can be used to express desired closed loop dynamics in a consistent way
- they can be implemented efficiently on a 14 DoF robot
- they work reliably for balancing and CoM tracking tasks despite model uncertainty, sensor noise and a naive estimation